جامعة الملك عبدالعزيز.

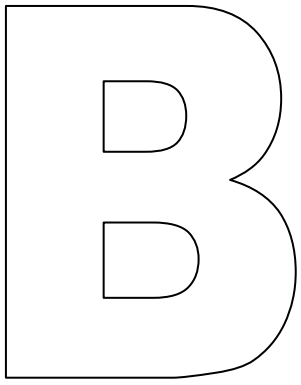
كلية العلوم. الاسم:_____

قسم الرياضيات. الرقم الجامعي:_____

math 202.
Calculus 2.

Final Exam

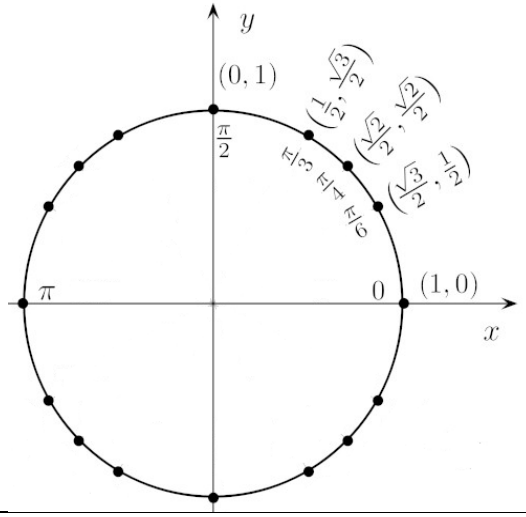
Date: Monday 30 / 6 / 1433 H.
Time: from 08:00 to 10:00.

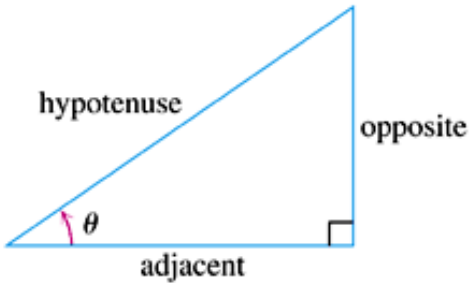


- تأكد من أن رمز نموذج الإجابة لديك هو B.
- أكتب اسمك على هذا النموذج ثم تأكد من تعبئة جميع بيانات نموذج الإجابة **خاصة رقمك الجامعي و بقلم الرصاص.**
- تأكد من تعبئة نموذج الحضور بصورة صحيحة.
- أجب عن جميع الأسئلة الآتية بتظليل الخيار الصحيح في نموذج الإجابة **بقلم الرصاص**.
- ممنوع استخدام الآلة الحاسبة.

هذه الصفحة تتضمن بعض القوانين التي قد تحتاجها لحل بعض أسئلة هذا الامتحان.

The Unit Circle





$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

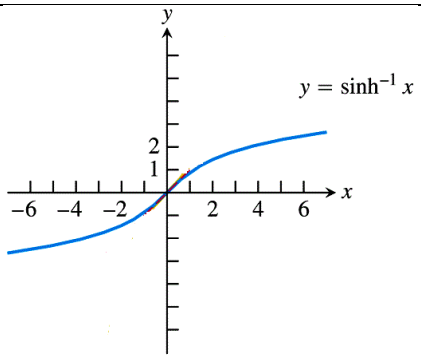
$$\frac{d}{dx}(a^x) = a^x \cdot \ln a$$

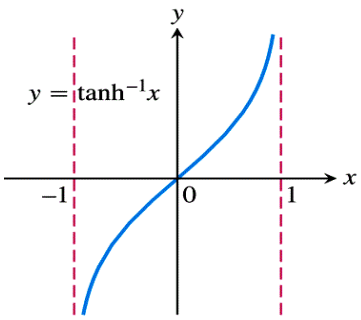
$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$





$$\int \sin^n x dx = -\tfrac{1}{n}\cos x\ \sin^{n-1}x + \tfrac{n-1}{n}\int \sin^{n-2}x\ dx \quad \text{where } n \in \mathbb{Z}, n \geq 2$$

$$\int \cos^n x dx = \tfrac{1}{n}\sin x\ \cos^{n-1}x + \tfrac{n-1}{n}\int \cos^{n-2}x\ dx \quad \text{where } n \in \mathbb{Z}, n \geq 2$$

$$\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

B

Q1. Definition $\cosh(-x) = -\cosh x$	
(A) TRUE	(B) FALSE

Q2. Definition The domain of the function $f(x) = \coth x$ is $(-\infty, 0) \cup (0, \infty)$.	
(A) TRUE	(B) FALSE

Q3. An antiderivative of $f(x) = \sqrt[3]{x^5}$ is $F(x) = \frac{5}{8}\sqrt[5]{x^8}$.	
(A) TRUE	(B) FALSE

Q4. Example 1, page A34. $1 + 3 + 9 + 27 + 81 =$				
(A) $\sum_{i=1}^{5} 3^i$	(B) $\sum_{i=0}^{5} \left(\frac{1}{3}\right)^i$	(C) $\sum_{i=0}^{5} 3^i$	(D) $\sum_{i=1}^{6} 3^i$	(E) $\sum_{i=0}^{4} 3^i$

Q5. If $f(x) \geq 0$ for each $x \in [a, b]$, then the definite integral $\int_a^b f(x)dx$ means the area of the region under the graph of f and the x-axis from the vertical line $x = a$ to the vertical line $x = b$.	
(A) TRUE	(B) FALSE

Q6. If $\int_1^3 f(x)dx = -2$ and $\int_1^3 g(x)dx = 4$, then $\int_1^3 [f(x) + 4g(x) + 5]\,dx =$				
(A) 21	(B) -18	(C) 18	(D) 24	(E) 20

Q7.

$$\int_{-\pi}^{\pi} \cos x \; dx \; > 2\pi$$

(A)	(B)
TRUE	FALSE

Q8. Problem 7, page 388.

If $g(x) = \int_1^x \frac{1}{t^2+1} \; dt$, then $g'(x) =$

(A)	(B)	(C)	(D)
$\frac{x^2+1}{2}$	$\frac{x}{\frac{x^3}{3}+x}$	$\frac{1}{x^2+1}$	$\frac{1}{t^2+1}+C$

Q9.

$$\int_{-9}^{9} \sinh^{-1} x \; dx \; = 0$$

(A)	(B)
TRUE	FALSE

Q10. Example 7, page 404

$$\int_0^4 \sqrt{2x+1} \; dx =$$

(A)	(B)	(C)	(D)	(E)
$\frac{29}{3}$	$\frac{26}{3}$	$\frac{23}{3}$	$\frac{20}{3}$	$\frac{17}{3}$

Q11.

$$\int \frac{2x^2 - \sqrt{x} \; + 1}{x} \; dx =$$

(A)	(B)	(C)
$2x^2 - \frac{2}{\sqrt{x}} + \frac{1}{x} + C$	$x^2 - \frac{2}{\sqrt{x}} + \frac{1}{x} + C$	$2x^2 - \frac{2}{\sqrt{x}} + \ln\lvert x\rvert + C$
(D)	(E)	
$x^2 - \frac{2}{\sqrt{x}} + \ln\lvert x\rvert + C$	$x^2 - 2\sqrt{x} + \ln\lvert x\rvert + C$	

Q12.

$$\int \sec(3x)\ \tan(3x)\,dx =$$

(A)	(B)	(C)	(D)
$\sec(3x) + C$	$-\sec(3x) + C$	$\frac{1}{3}\sec(3x) + C$	$-\frac{1}{3}\sec(3x) + C$

Q13.

$$\int (5^x + 5)\,dx =$$

(A)	(B)	(C)	(D)
$\frac{5^x}{\ln 5} + 5x + C$	$5^x \ln 5 + 5x + C$	$\frac{5^x}{\ln 5} + 5 + C$	$5^x + 5x + C$

Q14. Example 5, page 456

$$\int \tan^{-1} x\ dx =$$

Hint: $u = \tan^{-1} x$ and $dv = dx$

(A)	(B)	(C)
$x\tan^{-1} x + \frac{1}{2}\ln\lvert 1 + x^2\rvert + C$	$x\tan^{-1} x - \frac{1}{2}\ln\lvert 1 + x^2\rvert + C$	$x\tan^{-1} x - \ln\lvert 1 + x^2\rvert + C$
(D)	(E)	
$x\tan^{-1} x + \ln\lvert 1 + x^2\rvert + C$	$x\tan^{-1} x - \ln\left\lvert\frac{1 + x^2}{2}\right\rvert + C$	

السؤال رقم 15 هو تكرار للسؤال رقم 14 و يجب أن تجيب عليه للحصول على درجته

Q15. Example 5, page 456

$$\int \tan^{-1} x\ dx =$$

Hint: $u = \tan^{-1} x$ and $dv = dx$

(A)	(B)	(C)
$x\tan^{-1} x + \frac{1}{2}\ln\lvert 1 + x^2\rvert + C$	$x\tan^{-1} x - \frac{1}{2}\ln\lvert 1 + x^2\rvert + C$	$x\tan^{-1} x - \ln\lvert 1 + x^2\rvert + C$
(D)	(E)	
$x\tan^{-1} x + \ln\lvert 1 + x^2\rvert + C$	$x\tan^{-1} x - \ln\left\lvert\frac{1 + x^2}{2}\right\rvert + C$	

Q16.

$$\int \cos^{13}x \ dx =$$

Hint: Use a suitable formula from the second page.

(A) $\frac{1}{13}\cos x \sin^{12}x + \frac{12}{13}\int \sin^{11}x \ dx$

(B) $-\frac{1}{13}\sin x \cos^{12}x + \frac{12}{13}\int \cos^{11}x \ dx$

(C) $\frac{1}{13}\sin x \cos^{12}x + \frac{12}{13}\int \cos^{11}x \ dx$

(D) $-\frac{1}{13}\cos x \sin^{12}x + \frac{12}{13}\int \sin^{11}x \ dx$

Q17.

$$\int \frac{dx}{x(2+3x)^2} =$$

Hint: Use a suitable formula from the second page.

(A) $\frac{1}{2(2+3x)} - \frac{1}{9}\ln\left|\frac{2+3x}{x}\right| + C$

(B) $\frac{1}{2(2+3x)} - \frac{1}{4}\ln\left|\frac{2+3x}{x}\right| + C$

(C) $\frac{1}{2(2+3x)} - \ln\left|\frac{2+3x}{x}\right| + C$

(D) $\frac{1}{3(2+3x)} - \frac{1}{4}\ln\left|\frac{2+3x}{x}\right| + C$

Q18. Problem 4, page 481.

The form of the partial fraction decomposition of $\frac{2x+1}{(x+1)^2(x^2+4)^3}$ is

(A) $\frac{A}{(x+1)^2} + \frac{Bx+C}{(x^2+4)^3}$

(B) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{x^2+4} + \frac{E}{(x^2+4)^2}$

(C) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2} + \frac{Gx+H}{(x^2+4)^3}$

(D) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$

السؤال رقم 19 هو تكرار للسؤال رقم 18 و يجب أن تجيب عليه للحصول على درجته

Q19.

The form of the partial fraction decomposition of $\frac{2x+1}{(x+1)^2(x^2+4)^3}$ is

(A) $\frac{A}{(x+1)^2} + \frac{Bx+C}{(x^2+4)^3}$

(B) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{x^2+4} + \frac{E}{(x^2+4)^2}$

(C) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2} + \frac{Gx+H}{(x^2+4)^3}$

(D) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$

Q20. Problem 9, page 481

$$\int \frac{1}{x^2+3x-4}\,dx =$$

(A) $-\frac{1}{5}\ln\lvert x+4\rvert+\frac{1}{5}\ln\lvert x-1\rvert+C$	(B) $-\frac{1}{5}\ln\lvert x-4\rvert+\frac{1}{5}\ln\lvert x+1\rvert+C$
(C) $\frac{1}{5}\ln\lvert x+4\rvert-\frac{1}{5}\ln\lvert x-1\rvert+C$	(D) $\frac{1}{5}\ln\lvert x-4\rvert-\frac{1}{5}\ln\lvert x+1\rvert+C$

السؤال رقم 21 هو تكرار للسؤال رقم 20 و يجب أن تجيب عليه للحصول على درجته

Q21. Problem 9, page 481

$$\int \frac{1}{x^2+3x-4}\,dx =$$

(A) $-\frac{1}{5}\ln\lvert x+4\rvert+\frac{1}{5}\ln\lvert x-1\rvert+C$	(B) $-\frac{1}{5}\ln\lvert x-4\rvert+\frac{1}{5}\ln\lvert x+1\rvert+C$
(C) $\frac{1}{5}\ln\lvert x+4\rvert-\frac{1}{5}\ln\lvert x-1\rvert+C$	(D) $\frac{1}{5}\ln\lvert x-4\rvert-\frac{1}{5}\ln\lvert x+1\rvert+C$

Q22. Problem 1, page 465

$$\int \sin^3 x\ \cos^2 x\,dx =$$

(A) $\frac{\sin^5 x}{5}-\frac{\sin^3 x}{3}+C$	(B) $\frac{\cos^3 x}{3}-\frac{\cos^5 x}{5}+C$
(C) $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$	(D) $\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C$

السؤال رقم 23 هو تكرار للسؤال رقم 22 و يجب أن تجيب عليه للحصول على درجته

Q23.

$$\int \sin^3 x\ \cos^2 x\,dx =$$

(A) $\frac{\sin^5 x}{5}-\frac{\sin^3 x}{3}+C$	(B) $\frac{\cos^3 x}{3}-\frac{\cos^5 x}{5}+C$
(C) $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$	(D) $\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C$

Q24. Example 7, page 464 $\int \sec^3 x \, dx =$	
(A) $\frac{\tan x}{2} + \frac{1}{2} \ln\lvert\sec x + \tan x\rvert + C$	(B) $\frac{\sec x}{2} + \frac{1}{2} \ln\lvert\sec x + \tan x\rvert + C$
(C) $\frac{\sec x \tan x}{2} + \frac{1}{2} \ln\lvert\sec x + \tan x\rvert + C$	(D) $\frac{\sec^4 x}{4} + C$

السؤال رقم 25 هو تكرار للسؤال رقم 24 و يجب أن تجيب عليه للحصول على درجته

Q25. Example 7, page 464 $\int \sec^3 x \, dx =$	
(A) $\frac{\tan x}{2} + \frac{1}{2} \ln\lvert\sec x + \tan x\rvert + C$	(B) $\frac{\sec x}{2} + \frac{1}{2} \ln\lvert\sec x + \tan x\rvert + C$
(C) $\frac{\sec x \tan x}{2} + \frac{1}{2} \ln\lvert\sec x + \tan x\rvert + C$	(D) $\frac{\sec^4 x}{4} + C$

Q26. Problem 13, page 472 $\int \sqrt{1 - 4x^2} \, dx =$	
(A) $\frac{1}{2}\left[\sin^{-1}(4x) + 2x\sqrt{1 - 4x^2}\right] + C$	(B) $\frac{1}{4}\left[\sin^{-1}(2x) + 2x\sqrt{1 - 4x^2}\right] + C$
(C) $\frac{1}{2}\left[\sec^{-1}(4x) + 2x\sqrt{1 - 4x^2}\right] + C$	(D) $\frac{1}{4}\left[\sec^{-1}(2x) + 2x\sqrt{1 - 4x^2}\right] + C$

السؤال رقم 27 هو تكرار للسؤال رقم 26 و يجب أن تجيب عليه للحصول على درجته

Q27. Problem 13, page 472 $\int \sqrt{1-4x^2}\, dx =$	
(A) $\frac{1}{2}\left[\sin^{-1}(4x) + 2x\sqrt{1-4x^2}\ \right] + C$	(B) $\frac{1}{4}\left[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}\ \right] + C$
(C) $\frac{1}{2}\left[\sec^{-1}(4x) + 2x\sqrt{1-4x^2}\ \right] + C$	(D) $\frac{1}{4}\left[\sec^{-1}(2x) + 2x\sqrt{1-4x^2}\ \right] + C$

Q28. If $f(x)$ is continuous on $(-\infty, 3]$, then $\int_{-\infty}^{3} f(x)dx =$	
(A) $\lim_{b\to-\infty}\int_{b}^{3} f(x)dx$	(B) $\lim_{b\to\infty}\int_{3}^{b} f(x)dx$
(C) $\lim_{b\to3^-}\int_{3}^{b} f(x)dx$	(D) $\lim_{b\to3^+}\int_{3}^{b} f(x)dx$

Q29. $\int_{1}^{\infty} \frac{1}{x^{\sqrt{2}}}\, dx$ is	
(A) divergent	(B) convergent
(C) neither convergent nor divergent	

Q30. example 5, page 512 $\int_{1}^{3} \frac{1}{(x-1)^{\frac{2}{3}}}\, dx$	
(A) diverges	(B) converges to $\sqrt[3]{2}$

B

(C) converges to $3\sqrt[3]{2}$	(D) converges to 3

السؤال رقم 31 هو تكرار للسؤال رقم 30 و يجب أن تجيب عليه للحصول على درجته

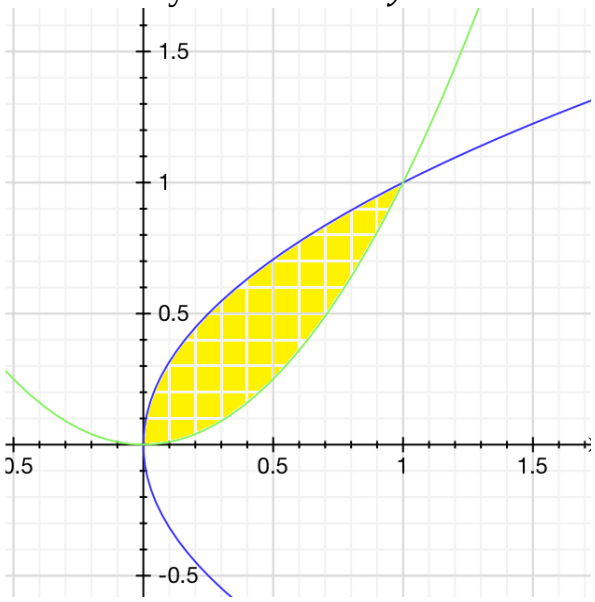
Q31. example 5, page 512

$$\int_1^3 \frac{1}{(x-1)^{\frac{2}{3}}}\,dx$$

(A) diverges	(B) converges to $\sqrt[3]{2}$
(C) converges to $3\sqrt[3]{2}$	(D) converges to 3

Q32. Problem 11, page 420.

The area of the region enclosed by the curves $y = x^2$ and $x = y^2$, is



(A)	(B)	(C)	(D)	(E)
$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$

السؤال رقم 33 هو تكرار للسؤال رقم 32 و يجب أن تجيب عليه للحصول على درجته

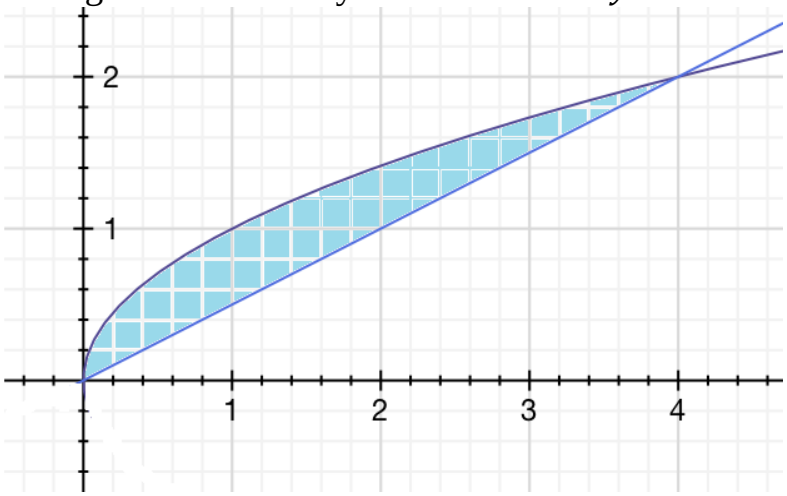
Q33. The area of the region enclosed by the curves $y = x^2$ and $x = y^2$, is

B

(A)	(B)	(C)	(D)	(E)
$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$

Q34. Problem 9, page 430.

By using the Washer Method, the volume of the solid generated by rotating about the y-axis the region bounded by the curve $x = y^2$ and the lines $x = 2y$ is



(A)	(B)	(C)	(D)	(E)
$\frac{61\pi}{5}$	$\frac{46\pi}{15}$	$\frac{4\pi}{3}$	$\frac{64\pi}{15}$	$\frac{21\pi}{5}$

السؤال رقم 35 هو تكرار للسؤال رقم 34 و يجب أن تجيب عليه للحصول على درجته

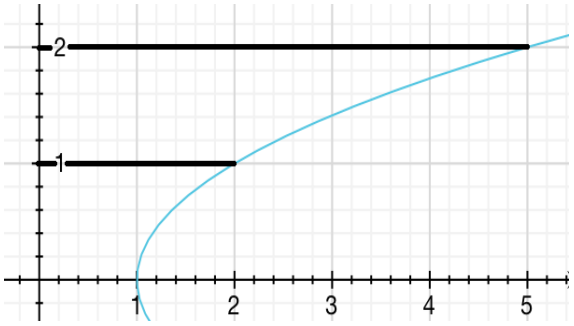
Q35.

By using the Washer Method, the volume of the solid generated by rotating about the y-axis the region bounded by the curve $x = y^2$ and the lines $x = 2y$ is

(A)	(B)	(C)	(D)	(E)
$\frac{61\pi}{5}$	$\frac{46\pi}{15}$	$\frac{4\pi}{3}$	$\frac{64\pi}{15}$	$\frac{21\pi}{5}$

Q36. Problem 10, page 436.

By using the Cylindrical Shell Method, the volume of the solid generated by rotating about the x-axis the region bounded by the curve $x = 1 + y^2$ and the lines $x = 0$, $y = 1$, and $y = 2$ is



(A)	(B)	(C)	(D)	(E)
$\frac{23\pi}{2}$	$\frac{21\pi}{2}$	$\frac{19\pi}{2}$	$\frac{17\pi}{2}$	$\frac{15\pi}{2}$

السؤال رقم 37 هو تكرار للسؤال رقم 36 و يجب أن تجيب عليه للحصول على درجته

Q37.

By using the Cylindrical Shell Method, the volume of the solid generated by rotating about the x-axis the region bounded by the curve $x = 1 + y^2$ and the lines $x = 0$, $y = 1$, and $y = 2$ is

(A)	(B)	(C)	(D)	(E)
$\frac{23\pi}{2}$	$\frac{21\pi}{2}$	$\frac{19\pi}{2}$	$\frac{17\pi}{2}$	$\frac{15\pi}{2}$

Q38. Problem 13, page 530.

The length of the curve $y = \ln(\cos x)$; where $0 \le x \le \frac{\pi}{3}$ is

(A)	(B)	(C)	(D)	(E)
$\ln(2+\sqrt{3})$	$\ln(1+\sqrt{3})$	$\ln(7-\sqrt{3})$	$\ln(2-\sqrt{3})$	$\ln(\sqrt{2}+3)$

السؤال رقم 39 هو تكرار للسؤال رقم 38 و يجب أن تجيب عليه للحصول على درجته

Q39.

The length of the curve $y = \ln(\cos x)$; where $0 \le x \le \frac{\pi}{3}$ is

(A)	(B)	(C)	(D)	(E)
$\ln(2+\sqrt{3})$	$\ln(1+\sqrt{3})$	$\ln(7-\sqrt{3})$	$\ln(2-\sqrt{3})$	$\ln(\sqrt{2}+3)$

Q40. Problem 4, page 537.

An integral for the area of the surface obtained by rotating the curve $x = \sqrt{y - y^2}$; $0 \le y \le 1$ about the x-axis is

(A) $\int_0^1 2\pi y^2 \sqrt{\frac{1}{y(1-y)}}\, dy$	(B) $\int_0^1 2\pi y \sqrt{\frac{1}{y(y-1)}}\, dy$	(C) $\int_0^1 2\pi y \sqrt{\frac{1}{y(1-y)}}\, dy$
(D) $\int_0^1 2\pi y \sqrt{\frac{1}{4y(y-1)}}\, dy$	(E) $\int_0^1 2\pi y \sqrt{\frac{1}{4y(1-y)}}\, dy$	

B

Answers to Final Exam

B

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
B	A	B	E	A	D	B	C	A	B	E	C	A	B	B	C	B	C	C	A

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	D	D	C	C	B	B	A	B	C	C	A	A	D	D	B	B	A	A	E