جامعة الملك عبدالعزيز.

كلية العلوم. الاسم:_____

قسم الرياضيات. الرقم الجامعي:_____

math 202.
Calculus 2.

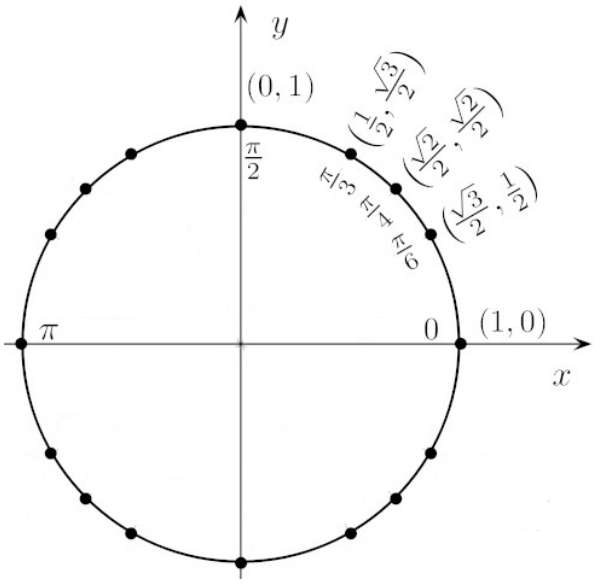
Final Exam

Date: Monday 8 / 2 / 1433 H.
Time: from 08:00 to 10:00.

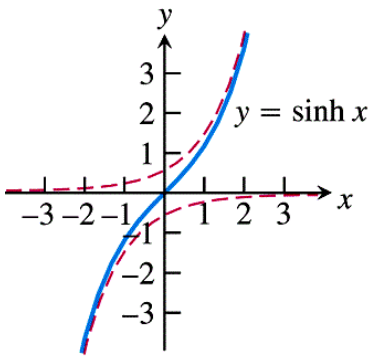
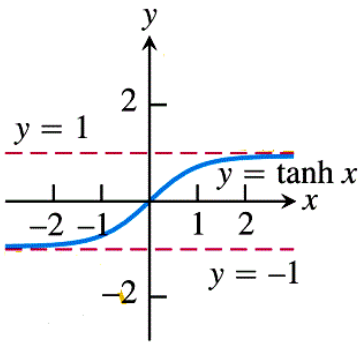
D

- تأكد من أن رمز نموذج الإجابة لديك هو D .
- أكتب اسمك على هذا النموذج ثم تأكد من تعبئة جميع بيانات نموذج الإجابة **خاصة رقمك الجامعي و بقلم الرصاص.**
- تأكد من تعبئة نموذج الحضور بصورة صحيحة.
- أجب عن جميع الأسئلة الآتية بتظليل الخيار الصحيح في نموذج الإجابة **بقلم الرصاص**.
- ممنوع استخدام الآلة الحاسبة.

هذه الصفحة تتضمن بعض القوانين التي قد تحتاجها لحل بعض أسئلة هذا الامتحان.



The Unit Circle

$\cosh^2 x - \sinh^2 x = 1$	$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
$\sinh(a+b) = \sinh a \cosh b + \cosh a \sinh b$	$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$
$\cosh(a+b) = \cosh a \cosh b + \sinh a \sinh b$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\displaystyle\int u \sin^{-1} u \; du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$	$\displaystyle\int u \cos^{-1} u \; du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$
	

Q1. $\lim_{x\to-\infty} \sinh x =$			
(A) $-\infty$	(B) ∞	(C) -1	(D) 1

Q2. $\coth^2 x =$			
(A) $1-\operatorname{csch}^2 x$	(B) $1-\operatorname{csch} x$	(C) $1+\operatorname{csch}^2 x$	(D) $1+\operatorname{csch} x$

Q3. If $y=\sqrt{2}\tanh^{-1}\sqrt{x}$, then $y'=\frac{dy}{dx}=$			
(A) $\frac{1}{2\sqrt{x}\ (1-x)}$	(B) $\frac{1}{2\sqrt{x}\ (1+x)}$	(C) $\frac{1}{\sqrt{x}\ (1-x)}$	(D) $\frac{1}{\sqrt{2x}\ (1-x)}$

Q4. $\int \frac{dx}{\sqrt{2x}-\sqrt{2}\sqrt{x^3}} =$ Hint: see question 3 above			
(A) $\frac{\tanh^{-1}\sqrt{x}}{2}+C$	(B) $\sqrt{2}\tanh^{-1}\sqrt{x}+C$	(C) $2\tanh^{-1}\sqrt{x}+C$	(D) $\frac{\tanh^{-1}\sqrt{x}}{\sqrt{2}}+C$

Q5. $\frac{d}{dx}\left(\frac{\cosh(x^2)}{4}\right)=$			
(A) $-x\ \sinh(x^2)$	(B) $x\ \sinh(x^2)$	(C) $\frac{-x\ \sinh(x^2)}{2}$	(D) $\frac{x\ \sinh(x^2)}{2}$

Q6. If $f'(x)=e^x+\frac{2}{\sqrt{1+x^2}}$ and $f(0)=-1$, then $f(x)=$			
(A) $e^x+2\sinh^{-1}x-3$	(B) $e^x+2\sinh^{-1}x-2$	(C) $e^x+2\cosh^{-1}x+3$	(D) $e^x+2\cosh^{-1}x+2$

Q7. If $\sum_{i=1}^{n} a_i=7$ and $\sum_{i=1}^{n} b_i=-13$, then $\sum_{i=1}^{n}(2a_i-2b_i)=$			
(A) 8	(B) 40	(C) 27	(D) -27

D

Q8.

The integral expression of $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\sin x_i}{x_i}\Delta x$ over the interval $[\pi, 2\pi]$ is

(A)	(B)	(C)	(D)
$\int_{\pi}^{2\pi}\frac{\sin x}{x}dx$	$\int_{\pi}^{2\pi}\frac{\cos x}{x}dx$	$\int_{\pi}^{2\pi}\frac{-\sin x}{x}dx$	$\int_{\pi}^{2\pi}\frac{-\cos x}{x}dx$

Q9.

If $\int_0^5 f(x)dx = -5$ and $\int_3^5 f(x)dx = 9$, then $\int_0^3 \frac{f(x)}{2}dx =$

(A)	(B)	(C)	(D)
2	-14	-7	$-\frac{5}{2}$

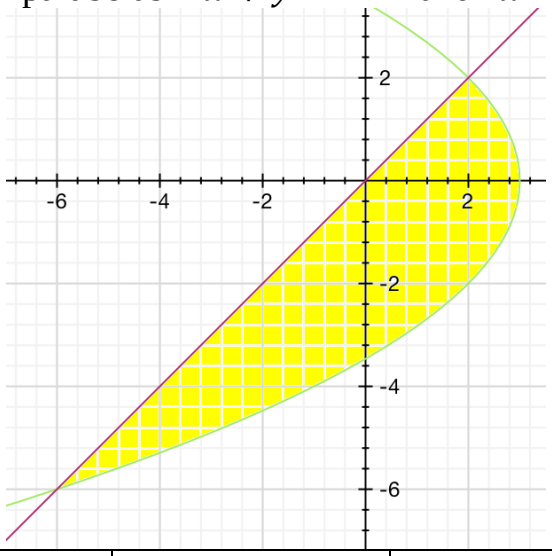
Q10.

$\frac{d}{dx}\left(\int_1^{x^3}\sin t\ dt\right) =$

(A)	(B)	(C)	(D)
$-3x^2\cos(x^3)$	$3x^2\cos(x^3)$	$-3x^2\sin(x^3)$	$3x^2\sin(x^3)$

Q11.

The area of the region enclosed by the parabolas $4x + y^2 = 12$ and $x = y$ is



(A)	(B)	(C)	(D)	(E)
$\frac{67}{3}$	$\frac{68}{3}$	$\frac{63}{3}$	$\frac{65}{3}$	$\frac{64}{3}$

السؤال رقم 12 هو تكرار للسؤال رقم 11 و يجب أن تجيب عليه للحصول على درجته

Q12.

The area of the region enclosed by the parabolas $4x + y^2 = 12$ and $x = y$ is

(A)	(B)	(C)	(D)	(E)
$\frac{67}{3}$	$\frac{68}{3}$	$\frac{63}{3}$	$\frac{65}{3}$	$\frac{64}{3}$

Q13.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{4}{3}\right)}}$ over the interval $[1, \infty)$ is

(A)	(B)	(C)	(D)	(E)
2	3	4	5	6

السؤال رقم 14 هو تكرار للسؤال رقم 13 و يجب أن تجيب عليه للحصول على درجته

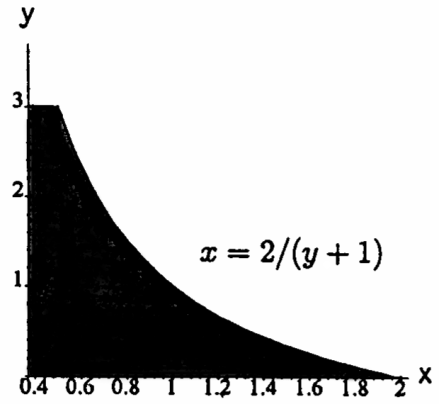
Q14.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{4}{3}\right)}}$ over the interval $[1, \infty)$ is

(A)	(B)	(C)	(D)	(E)
2	3	4	5	6

Q15.

The volume of solid generated by rotating the region bounded by curve $x = \frac{2}{y+1}$ and the lines $x = 0$, $y = 0$, and $y = 3$, about the y-axis is



(A)	(B)	(C)	(D)	(E)
$\frac{11\pi}{2}$	$\frac{9\pi}{2}$	4π	3π	2π

السؤال رقم 16 هو تكرار للسؤال رقم 15 و يجب أن تجيب عليه للحصول على درجته

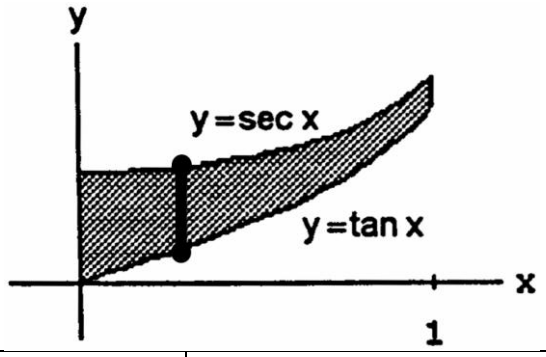
Q16.

The volume of solid generated by rotating the region bounded by curve $x = \frac{2}{y+1}$ and the lines $x = 0$, $y = 0$, and $y = 3$, about the y-axis is

(A)	(B)	(C)	(D)	(E)
$\frac{11\pi}{2}$	$\frac{9\pi}{2}$	4π	3π	2π

Q17.

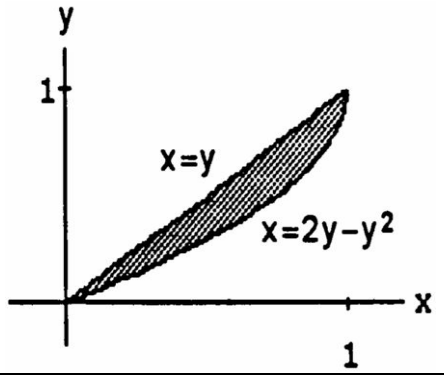
The integral which gives the volume of the solid generated by rotating about the x-axis the region bounded by the curves $y = \sec x$ and $y = \tan x$ and the lines $x = 0$ and $x = 1$ is



(A) $V = \pi \int_0^1 [\sec^2 x \ - \ \tan^2 x]\, dx$	(B) $V = \pi \int_0^1 [\tan^2 x \ - \ \sec^2 x]\, dx$
(C) $V = \int_0^1 [\sec^2 x \ - \ \tan^2 x]\, dx$	(D) $V = \int_0^1 [\tan^2 x \ - \ \sec^2 x]\, dx$

Q18.

By using the Shell Method, the integral which gives the volume of the solid generated by rotating about the x-axis the region bounded by the curve $x = 2y - y^2$ and the lines $x = y$ is



(A) $V = \int_0^1 \ 2\pi\, y \ [y^2 \ - 3y]\, dy$	(B) $V = \int_0^1 \ 2\pi\, y \ [y^2 \ - y]\, dy$
(C) $V = \int_0^1 \ 2\pi\, y \ [3y - y^2]\, dy$	(D) $V = \int_0^1 \ 2\pi\, y \ [y - y^2]\, dy$

Q19.

$\int_0^3 \frac{dx}{x-1} =$

(A)	(B)	(C)	(D)	(E)
$\ln 2$	divergent	$\ln 3$	$\ln \frac{2}{3}$	$\ln 4$

Q20.

$$\int_0^1 \log_8 x \; dx =$$

(A)	(B)	(C)	(D)	(E)
$-\frac{1}{\ln 8}$	$\frac{1}{\ln 8}$	divergent	$\ln 8$	8

Q21.

$$\int \frac{dx}{\sqrt{4x-x^2}} =$$

Hint: complete the square then use a suitable formula.

(A)	(B)	(C)	(D)	(E)
$\frac{\ln(4-2x)}{\sqrt{4x-x^2}} + C$	$\frac{4-2x}{\sqrt{4x-x^2}} + C$	$\sin^{-1}\left(\frac{x-2}{2}\right) + C$	$\tan^{-1}\left(\frac{x-1}{2}\right) + C$	$\sin^{-1}\left(\frac{x-1}{2}\right) + C$

السؤال رقم 22 هو تكرار للسؤال رقم 21 و يجب أن تجيب عليه للحصول على درجته

Q22.

$$\int \frac{dx}{\sqrt{4x-x^2}} =$$

Hint: complete the square then use a suitable formula.

(A)	(B)	(C)	(D)	(E)
$\frac{\ln(4-2x)}{\sqrt{4x-x^2}} + C$	$\frac{4-2x}{\sqrt{4x-x^2}} + C$	$\sin^{-1}\left(\frac{x-2}{2}\right) + C$	$\tan^{-1}\left(\frac{x-1}{2}\right) + C$	$\sin^{-1}\left(\frac{x-1}{2}\right) + C$

Q23.

$$\int x^8 \ln x \; dx =$$

(A)	(B)	(C)	(D)	(E)
$\frac{x^8 \ln x}{81} - \frac{x^9}{9} + C$	$\frac{x^8 \ln x}{9} - \frac{x^9}{9} + C$	$\frac{x^8 \ln x}{9} - \frac{x^9}{81} + C$	$\frac{x^9 \ln x}{9} - \frac{x^9}{81} + C$	$\frac{x^9 \ln x}{9} + \frac{x^9}{81} + C$

السؤال رقم 24 هو تكرار للسؤال رقم 23 و يجب أن تجيب عليه للحصول على درجته

Q24.

$$\int x^8 \ln x \; dx =$$

(A)	(B)	(C)	(D)	(E)
$\frac{x^8 \ln x}{81} - \frac{x^9}{9} + C$	$\frac{x^8 \ln x}{9} - \frac{x^9}{9} + C$	$\frac{x^8 \ln x}{9} - \frac{x^9}{81} + C$	$\frac{x^9 \ln x}{9} - \frac{x^9}{81} + C$	$\frac{x^9 \ln x}{9} + \frac{x^9}{81} + C$

Q25.

$$\int \cos^2 x \tan^3 x \; dx =$$

(A)	(B)	(C)
$\frac{\cos^2 x}{2} + \ln\lvert\cos x\rvert + C$	$\frac{\cos^2 x}{2} - \ln\lvert\cos x\rvert + C$	$-\frac{\cos^2 x}{2} - \ln\lvert\cos x\rvert + C$
(D) $\ln\lvert\cos x\rvert - \frac{\cos^2 x}{2} + C$	(E) $\frac{\cos^2 x - \ln\lvert\cos x\rvert}{2} + C$	

السؤال رقم 26 هو تكرار للسؤال رقم 25 و يجب أن تجيب عليه للحصول على درجته

Q26.

$$\int \cos^2 x \tan^3 x \, dx =$$

(A) $\frac{\cos^2 x}{2} + \ln\lvert\cos x\rvert + C$	(B) $\frac{\cos^2 x}{2} - \ln\lvert\cos x\rvert + C$	(C) $-\frac{\cos^2 x}{2} - \ln\lvert\cos x\rvert + C$
(D) $\ln\lvert\cos x\rvert - \frac{\cos^2 x}{2} + C$	(E) $\frac{\cos^2 x - \ln\lvert\cos x\rvert}{2} + C$	

Q27.

$$\int \sqrt{1-25x^2}\ dx =$$ Hint: let $5x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(A) $\frac{1}{10}\left[\sin^{-1}(25x^2) + x\sqrt{1-25x^2}\right] + C$	(B) $\frac{1}{10}\left[\sin^{-1}(25x^2) + 5x\sqrt{1-25x^2}\right] + C$	(C) $\frac{1}{10}\left[\sin^{-1}(5x) + 5\sqrt{1-25x^2}\right] + C$
(D) $\frac{1}{10}\left[\sin^{-1}(5x) + x\sqrt{1-25x^2}\right] + C$	(E) $\frac{1}{10}\left[\sin^{-1}(5x) + 5x\sqrt{1-25x^2}\right] + C$	

السؤال رقم 28 هو تكرار للسؤال رقم 27 و يجب أن تجيب عليه للحصول على درجته

Q28.

$$\int \sqrt{1-25x^2}\ dx =$$ Hint: let $5x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(A) $\frac{1}{10}\left[\sin^{-1}(25x^2) + x\sqrt{1-25x^2}\right] + C$	(B) $\frac{1}{10}\left[\sin^{-1}(25x^2) + 5x\sqrt{1-25x^2}\right] + C$	(C) $\frac{1}{10}\left[\sin^{-1}(5x) + 5\sqrt{1-25x^2}\right] + C$
(D) $\frac{1}{10}\left[\sin^{-1}(5x) + x\sqrt{1-25x^2}\right] + C$	(E) $\frac{1}{10}\left[\sin^{-1}(5x) + 5x\sqrt{1-25x^2}\right] + C$	

Q29.

Using the substitution way with $u = \tan(2\theta)$, the evaluation of the integral $\int \sec^2(2\theta)\ \tan(2\theta)\ d\theta$ is

(A) $\frac{1}{4}\cot^2(2\theta) + C$	(B) $-\frac{1}{4}\tan^2(2\theta) + C$	(C) $\frac{1}{4}\tan^2(2\theta) + C$	(D) $-\frac{1}{4}\cot^2(2\theta) + C$

Q30.

Using the substitution way with $u = \sec(2\theta)$, the evaluation of the integral $\int \sec^2(2\theta)\ \tan(2\theta)\ d\theta$ is

(A) $-\frac{1}{4}\sec^2(2\theta) + C$	(B) $\frac{1}{4}\sec^2(2\theta) + C$	(C) $-\frac{1}{4}\csc^2(2\theta) + C$	(D) $\frac{1}{4}\csc^2(2\theta) + C$

Q31.

$\int \frac{\cos^{-1}\sqrt{x}}{2}\, dx =$ Hint: use a suitable formula.

(A) $\frac{(2\sqrt{x}-1)\cos^{-1}\sqrt{x} - x\sqrt{1-x}}{4} + C$	(B) $\frac{(2\sqrt{x}-1)\cos^{-1}\sqrt{x} - \sqrt{x-x^2}}{4} + C$
(C) $\frac{(2x-1)\cos^{-1}\sqrt{x} - x\sqrt{1-x}}{4} + C$	(D) $\frac{(2x-1)\cos^{-1}\sqrt{x} - \sqrt{x-x^2}}{4} + C$

Q32.

If $\frac{6x^2-23x+5}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$, then

(A) $A=2, B=5, C=-1$	(B) $A=-2, B=-5, C=1$	(C) $A=-2, B=5, C=1$	(D) $A=2, B=-5, C=-1$

Q33.

$\int \frac{6x^2-23x+5}{(x-1)(x+2)(x-3)}\, dx =$

(A) $-2\ln\lvert x-1\rvert - 5\ln\lvert x+2\rvert + \ln\lvert x-3\rvert + C$	(B) $2\ln\lvert x-1\rvert - 5\ln\lvert x+2\rvert - \ln\lvert x-3\rvert + C$
(C) $-2\ln\lvert x-1\rvert + 5\ln\lvert x+2\rvert + \ln\lvert x-3\rvert + C$	(D) $2\ln\lvert x-1\rvert + 5\ln\lvert x+2\rvert - \ln\lvert x-3\rvert + C$

Q34.

If f is continuous on $(-\infty, 1) \cup (1, 2]$ and discontinuous at 1, then $\int_{-\infty}^{2} f(x)dx =$

(A) $\lim_{c\to-\infty} \int_c^0 f(x)dx + \lim_{b\to1^+} \int_0^b f(x)dx + \lim_{b\to1^-} \int_b^2 f(x)dx$
(B) $\lim_{c\to-\infty} \int_{-4}^0 cf(x)dx + \lim_{b\to1^+} \int_0^b f(x)dx + \lim_{b\to1^-} \int_b^2 f(x)dx$
(C) $\lim_{c\to-\infty} \int_c^0 f(x)dx + \lim_{b\to1^-} \int_0^b f(x)dx + \lim_{b\to1^+} \int_b^2 f(x)dx$
(D) $\lim_{c\to-\infty} \int_{-4}^0 cf(x)dx + \lim_{b\to1^-} \int_0^b f(x)dx + \lim_{b\to1^+} \int_b^2 f(x)dx$

Q35.

The integral $\int_1^{\infty} \frac{2+e^{-x}}{x}\, dx$ is

(A) convergent and divergent	(B) convergent	(C) neither convergent nor divergent	(D) divergent

السؤال رقم 36 هو تكرار للسؤال رقم 35 و يجب أن تجيب عليه للحصول على درجته

Q36.

The integral $\int_1^{\infty} \frac{2+e^{-x}}{x}\, dx$ is

(A) convergent and divergent	(B) convergent	(C) neither convergent nor divergent	(D) divergent

Q37. The length of the curve $y = \ln(\cos x);\ 0 \le x \le \frac{\pi}{3}$, is				
(A) $\frac{2+\sqrt{3}}{\pi}$	(B) $(2+\sqrt{3})\pi$	(C) $\ln(1+\sqrt{3})$	(D) $\ln(3+\sqrt{2})$	(E) $\ln(2+\sqrt{3})$

السؤال رقم 38 هو تكرار للسؤال رقم 37 و يجب أن تجيب عليه للحصول على درجته

Q38. The length of the curve $y = \ln(\cos x);\ 0 \le x \le \frac{\pi}{3}$, is				
(A) $\frac{2+\sqrt{3}}{\pi}$	(B) $(2+\sqrt{3})\pi$	(C) $\ln(1+\sqrt{3})$	(D) $\ln(3+\sqrt{2})$	(E) $\ln(2+\sqrt{3})$

Q39. The curve $x = \sqrt{4-y^2}$, where $0 \le y \le 1$, is rotated about the y-axis. The area of the resulting surface is				
(A) 4π	(B) 42π	(C) 12π	(D) 164π	(E) 146π

السؤال رقم 40 هو تكرار للسؤال رقم 39 و يجب أن تجيب عليه للحصول على درجته

Q40. The curve $x = \sqrt{4-y^2}$, where $0 \le y \le 1$, is rotated about the y-axis. The area of the resulting surface is				
(A) 4π	(B) 42π	(C) 12π	(D) 164π	(E) 146π