جامعة الملك عبدالعزيز.

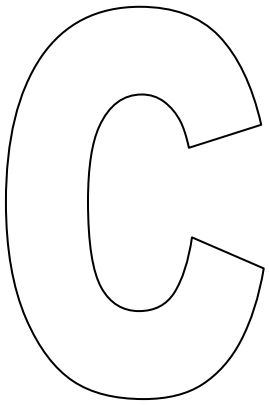
كلية العلوم. الاسم:_____

قسم الرياضيات. الرقم الجامعي:_____

math 202.
Calculus 2.

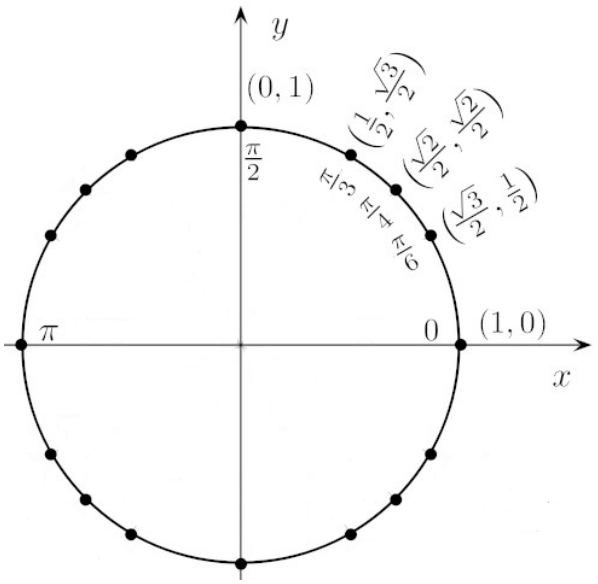
Final Exam

Date: Monday 8 / 2 / 1433 H.
Time: from 08:00 to 10:00.

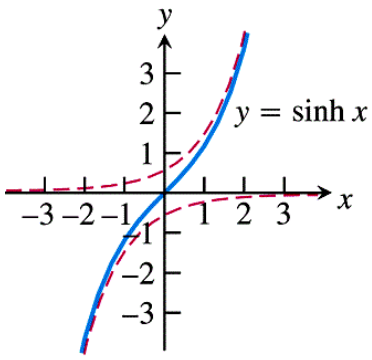
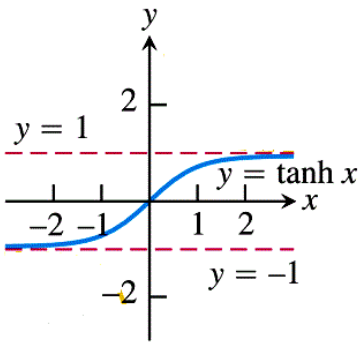


- تأكد من أن رمز نموذج الإجابة لديك هو C .
- أكتب اسمك على هذا النموذج ثم تأكد من تعبئة جميع بيانات نموذج الإجابة **خاصة رقمك الجامعي و بقلم الرصاص.**
- تأكد من تعبئة نموذج الحضور بصورة صحيحة.
- أجب عن جميع الأسئلة الآتية بتظليل الخيار الصحيح في نموذج الإجابة **بقلم الرصاص**.
- ممنوع استخدام الآلة الحاسبة.

هذه الصفحة تتضمن بعض القوانين التي قد تحتاجها لحل بعض أسئلة هذا الامتحان.



The Unit Circle

$\cosh^2 x - \sinh^2 x = 1$	$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
$\sinh(a+b) = \sinh a \cosh b + \cosh a \sinh b$	$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$
$\cosh(a+b) = \cosh a \cosh b + \sinh a \sinh b$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\displaystyle\int u \sin^{-1} u \; du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$	$\displaystyle\int u \cos^{-1} u \; du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$
	

Q1. $\lim_{x\to-\infty} \tanh x =$			
(A) ∞	(B) $-\infty$	(C) 1	(D) -1

Q2. $\cosh 2x =$			
(A) $\cosh^2 x + \sinh^2 x$	(B) $\cosh x + \sinh^2 x$	(C) $\cosh^2 x + \sinh x$	(D) $\cosh x + \sinh x$

Q3. If $y = 4\tanh^{-1}\sqrt{x}$, then $y' = \frac{dy}{dx} =$			
(A) $\frac{5}{2\sqrt{x}\ (1+x)}$	(B) $\frac{2}{\sqrt{x}\ (1-x)}$	(C) $\frac{1}{\sqrt{x}\ (1-x)}$	(D) $\frac{3}{\sqrt{x}\ (1-x)}$

Q4. $\int \frac{2dx}{\sqrt{x}-\sqrt{x^3}} =$ Hint: see question 3 above			
(A) $\tanh^{-1}\sqrt{x} + C$	(B) $2\tanh^{-1}\sqrt{x} + C$	(C) $4\tanh^{-1}\sqrt{x} + C$	(D) $\frac{\tanh^{-1}\sqrt{x}}{2} + C$

Q5. $\frac{d}{dx}(3\cosh(x^2)) =$			
(A) $-6x\ \sinh(x^2)$	(B) $6x\ \sinh(x^2)$	(C) $-6\ \sinh(x^2)$	(D) $6\ \sinh(x^2)$

Q6. If $f'(x) = e^x + \frac{4}{\sqrt{1+x^2}}$ and $f(0) = 9$, then $f(x) =$			
(A) $e^x + 4\sinh^{-1} x + 8$	(B) $e^x + 4\sinh^{-1} x + 10$	(C) $e^x + 4\cosh^{-1} x + 8$	(D) $e^x + 4\cosh^{-1} x + 10$

Q7. If $\sum_{i=1}^{n} a_i = 7$ and $\sum_{i=1}^{n} b_i = -13$, then $\sum_{i=1}^{n}(b_i - 2a_i) =$			
(A) 8	(B) 40	(C) 27	(D) -27

Q8.

The integral expression of $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \ln(1+x_i^4)\,\Delta x$ over the interval $[\,1\,,\,5\,]$ is

(A)	(B)	(C)	(D)
$\int_1^5 \frac{x}{1+x^4}\,dx$	$\int_1^5 x\ln(1+x^4)\,dx$	$\int_1^5 x(1+4)\,dx$	$\int_1^5 x\ln(x^4)\,dx$

Q9.

If $\int_2^8 f(x)dx = 21$ and $\int_5^8 f(x)dx = -3$, then $\int_2^5 \frac{f(x)}{8}dx =$

(A)	(B)	(C)	(D)
24	-3	21	3

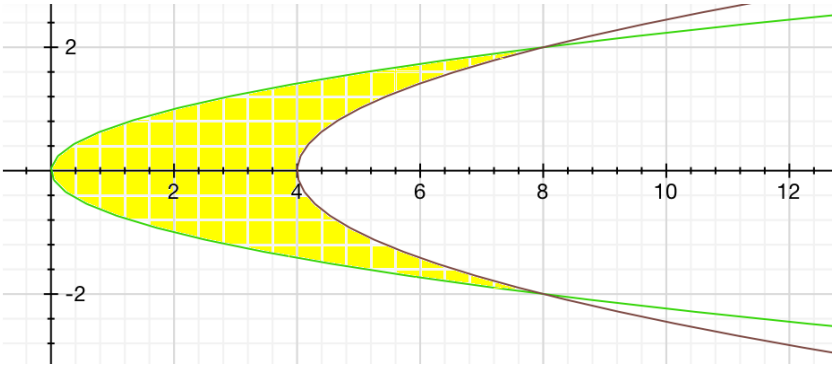
Q10.

$\frac{d}{dx}\left(\int_1^{x^4} \sec t\ dt\right) =$

(A)	(B)	(C)	(D)
$\sec(x^4)\tan(x^4)$	$\frac{\sec(x^4)\tan(x^4)}{4x^3}$	$4x^3\sec(x^4)$	$4x^3\sec\left(\frac{x^5}{5}\right)$

Q11.

The area of the region enclosed by the parabolas $x = 2y^2$ and $x = 4 + y^2$ is



(A)	(B)	(C)	(D)	(E)
$\frac{28}{3}$	$\frac{26}{3}$	$\frac{31}{3}$	$\frac{32}{3}$	$\frac{29}{3}$

السؤال رقم 12 هو تكرار للسؤال رقم 11 و يجب أن تجيب عليه للحصول على درجته

Q12.

The area of the region enclosed by the parabolas $x = 2y^2$ and $x = 4 + y^2$ is

(A)	(B)	(C)	(D)	(E)
$\frac{28}{3}$	$\frac{26}{3}$	$\frac{31}{3}$	$\frac{32}{3}$	$\frac{29}{3}$

C

Q13.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{5}{4}\right)}}$ over the interval $[1, \infty)$ is

(A)	(B)	(C)	(D)	(E)
2	3	4	5	6

السؤال رقم 14 هو تكرار للسؤال رقم 13 و يجب أن تجيب عليه للحصول على درجته

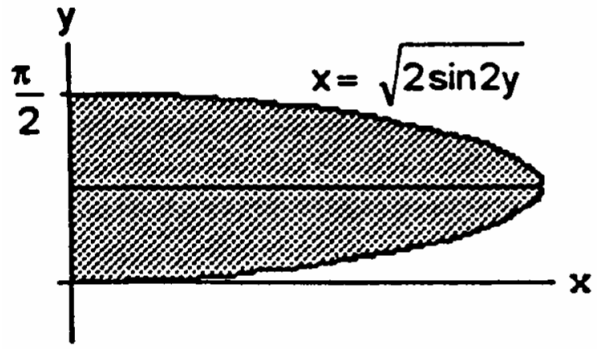
Q14.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{5}{4}\right)}}$ over the interval $[1, \infty)$ is

(A)	(B)	(C)	(D)	(E)
2	3	4	5	6

Q15.

The volume of solid generated by rotating the region bounded by curve $x = \sqrt{2 \sin 2y}$, where $0 \leq y \leq \frac{\pi}{2}$ and the line $x = 0,$, about the y-axis is



(A)	(B)	(C)	(D)	(E)
4π	$\frac{7\pi}{2}$	3π	$\frac{5\pi}{2}$	2π

السؤال رقم 16 هو تكرار للسؤال رقم 15 و يجب أن تجيب عليه للحصول على درجته

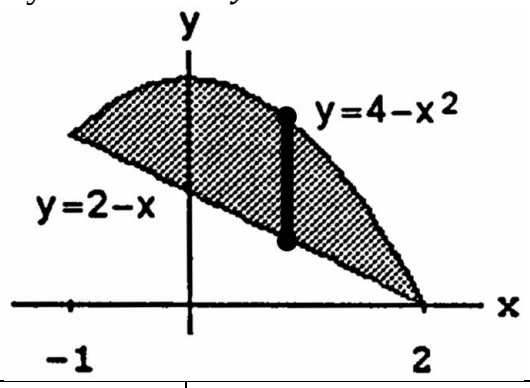
Q16.

The volume of solid generated by rotating the region bounded by curve $x = \sqrt{2 \sin 2y}$, where $0 \leq y \leq \frac{\pi}{2}$ and the line $x = 0,$, about the y-axis is

(A)	(B)	(C)	(D)	(E)
4π	$\frac{7\pi}{2}$	3π	$\frac{5\pi}{2}$	2π

Q17.

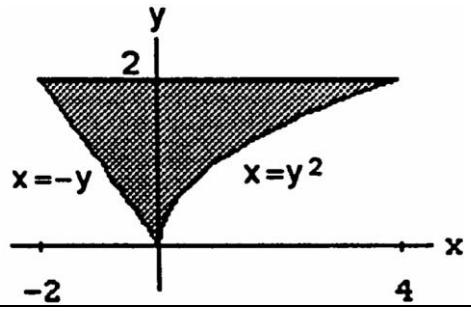
The integral which gives the volume of the solid generated by rotating about the x-axis the region bounded by the curve $y = 4 - x^2$ and the line $y = 2 - x$ is



(A) $V = \pi \int_0^4 [(2-x)^2 - (4-x^2)^2]\, dx$	(B) $V = \pi \int_0^4 [(4-x^2)^2 - (2-x)^2]\, dx$
(C) $V = \pi \int_{-1}^2 [(2-x)^2 - (4-x^2)^2]\, dx$	(D) $V = \pi \int_{-1}^2 [(4-x^2)^2 - (2-x)^2]\, dx$

Q18.

By using the Shell Method, the integral which gives the volume of the solid generated by rotating about the x-axis the region bounded by the curve $x = y^2$ where $y \geq 0$ and the lines $x = -y$ and $y = 2$ is



(A) $V = \int_0^2 2\pi y[y^2 + (-y)]\, dy$	(B) $V = \int_{-2}^4 2\pi y[y^2 - (-y)]\, dy$
(C) $V = \int_0^2 2\pi y[y^2 - (-y)]\, dy$	(D) $V = \int_{-2}^4 2\pi y[y^2 + (-y)]\, dy$

Q19.

$\int_0^4 \frac{dx}{x-1} =$

(A)	(B)	(C)	(D)	(E)
$\ln 3$	$\ln 4$	$\ln 2$	$\ln \frac{3}{2}$	divergent

Q20.

$\int_0^1 \log_7 x\ dx =$

(A)	(B)	(C)	(D)	(E)
$\ln 7$	7	divergent	$-\frac{1}{\ln 7}$	$\frac{1}{\ln 7}$

Q21.

$$\int \frac{dx}{\sqrt{8+2x-x^2}} =$$

Hint: complete the square then use a suitable formula.

(A)	(B)	(C)	(D)	(E)
$\sin^{-1}\left(\frac{x-1}{3}\right)+C$	$\sin^{-1}\left(\frac{x-1}{2}\right)+C$	$\frac{2-2x}{\sqrt{8+2x-x^2}}+C$	$\frac{\ln(2-2x)}{\sqrt{8+2x-x^2}}+C$	$\tan^{-1}\left(\frac{x-1}{2}\right)+C$

السؤال رقم 22 هو تكرار للسؤال رقم 21 و يجب أن تجيب عليه للحصول على درجته

Q22.

$$\int \frac{dx}{\sqrt{8+2x-x^2}} =$$

(A)	(B)	(C)	(D)	(E)
$\sin^{-1}\left(\frac{x-1}{3}\right)+C$	$\sin^{-1}\left(\frac{x-1}{2}\right)+C$	$\frac{2-2x}{\sqrt{8+2x-x^2}}+C$	$\frac{\ln(2-2x)}{\sqrt{8+2x-x^2}}+C$	$\tan^{-1}\left(\frac{x-1}{2}\right)+C$

Q23.

$$\int x^6 \ln x \ dx =$$

(A)	(B)	(C)	(D)	(E)
$\frac{x^7 \ln x}{7}-\frac{x^7}{7}+C$	$\frac{x^7 \ln x}{7}-\frac{x^7}{49}+C$	$\frac{x^6 \ln x}{7}-\frac{x^7}{49}+C$	$\frac{x^6 \ln x}{7}-\frac{x^7}{7}+C$	$\frac{x^7 \ln x}{7}+\frac{x^7}{49}+C$

السؤال رقم 24 هو تكرار للسؤال رقم 23 و يجب أن تجيب عليه للحصول على درجته

Q24.

$$\int x^6 \ln x \ dx =$$

(A)	(B)	(C)	(D)	(E)
$\frac{x^7 \ln x}{7}-\frac{x^7}{7}+C$	$\frac{x^7 \ln x}{7}-\frac{x^7}{49}+C$	$\frac{x^6 \ln x}{7}-\frac{x^7}{49}+C$	$\frac{x^6 \ln x}{7}-\frac{x^7}{7}+C$	$\frac{x^7 \ln x}{7}+\frac{x^7}{49}+C$

Q25.

$$\int \cot^5\theta \sin^4\theta \ d\theta =$$

(A)	(B)	(C)
$\ln\lvert\sin\theta\rvert-\sin^2\theta+\frac{\sin^3\theta}{3}+C$	$\ln\lvert\sin\theta\rvert+\sin^2\theta+\frac{\sin^4\theta}{4}+C$	$2\ln\lvert\sin\theta\rvert-\sin^2\theta+\frac{\sin^3\theta}{3}+C$
(D)	(E)	
$\ln\lvert\sin\theta\rvert+\sin^2\theta+\frac{\sin^3\theta}{3}+C$	$\ln\lvert\sin\theta\rvert-\sin^2\theta+\frac{\sin^4\theta}{4}+C$	

السؤال رقم 26 هو تكرار للسؤال رقم 25 و يجب أن تجيب عليه للحصول على درجته

Q26.

$$\int \cot^5\theta \sin^4\theta \ d\theta =$$

(A)	(B)	(C)
$\ln\lvert\sin\theta\rvert-\sin^2\theta+\frac{\sin^3\theta}{3}+C$	$\ln\lvert\sin\theta\rvert+\sin^2\theta+\frac{\sin^4\theta}{4}+C$	$2\ln\lvert\sin\theta\rvert-\sin^2\theta+\frac{\sin^3\theta}{3}+C$
(D)	(E)	
$\ln\lvert\sin\theta\rvert+\sin^2\theta+\frac{\sin^3\theta}{3}+C$	$\ln\lvert\sin\theta\rvert-\sin^2\theta+\frac{\sin^4\theta}{4}+C$	

Q27.

$$\int \sqrt{1-16x^2}\ dx =$$ Hint: let $4x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(A) $\frac{1}{8}\left[\sin^{-1}(16x^2) + 8x\sqrt{1-16x^2}\right] + C$	(B) $\frac{1}{8}\left[\sin^{-1}(16x^2) + 4x\sqrt{1-16x^2}\right] + C$	(C) $\frac{1}{8}\left[\sin^{-1}(4x) + 4\sqrt{1-16x^2}\right] + C$
(D) $\frac{1}{8}\left[\sin^{-1}(4x) + 4x\sqrt{1-16x^2}\right] + C$	(E) $\frac{1}{8}\left[\sin^{-1}(4x) + x\sqrt{1-16x^2}\right] + C$	

السؤال رقم 28 هو تكرار للسؤال رقم 27 و يجب أن تجيب عليه للحصول على درجته

Q28.

$$\int \sqrt{1-16x^2}\ dx =$$ Hint: let $4x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(A) $\frac{1}{8}\left[\sin^{-1}(16x^2) + 8x\sqrt{1-16x^2}\right] + C$	(B) $\frac{1}{8}\left[\sin^{-1}(16x^2) + 4x\sqrt{1-16x^2}\right] + C$	(C) $\frac{1}{8}\left[\sin^{-1}(4x) + 4\sqrt{1-16x^2}\right] + C$
(D) $\frac{1}{8}\left[\sin^{-1}(4x) + 4x\sqrt{1-16x^2}\right] + C$	(E) $\frac{1}{8}\left[\sin^{-1}(4x) + x\sqrt{1-16x^2}\right] + C$	

Q29.

Using the substitution way with $u = \cot(2\theta)$, the evaluation of the integral $\int 4\csc^2(2\theta)\cot(2\theta)\ d\theta$ is

(A) $\cot^2(2\theta) + C$	(B) $-\cot^2(2\theta) + C$	(C) $\tan^2(2\theta) + C$	(D) $-\tan^2(2\theta) + C$

Q30.

Using the substitution way with $u = \csc(2\theta)$, the evaluation of the integral $\int 4\csc^2(2\theta)\cot(2\theta)\ d\theta$ is

(A) $-\csc^2(2\theta) + C$	(B) $\sec^2(2\theta) + C$	(C) $-\sec^2(2\theta) + C$	(D) $\csc^2(2\theta) + C$

Q31.

$$\int 2\sin^{-1}\sqrt{x}\ dx =$$ Hint: use a suitable formula.

(A) $(2x-1)\sin^{-1}\sqrt{x} + x\sqrt{1-x} + C$	(B) $\left(2\sqrt{x}-1\right)\sin^{-1}\sqrt{x} + x\sqrt{1-x} + C$
(C) $(2x-1)\sin^{-1}\sqrt{x} + \sqrt{x-x^2} + C$	(D) $\left(2\sqrt{x}-1\right)\sin^{-1}\sqrt{x} + \sqrt{x-x^2} + C$

Q32.

If $\frac{2x^2+10x+14}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$, then

(A)	(B)	(C)	(D)
$A=3, B=-2, C=-1$	$A=3, B=-2, C=1$	$A=-3, B=-2, C=1$	$A=-3, B=-2, C=-1$

Q33.

$\int \frac{2x^2+10x+14}{(x+1)(x+2)(x+3)}\, dx =$

(A)	(B)
$3\ln\lvert x+1\rvert - 2\ln\lvert x+2\rvert - \ln\lvert x+3\rvert + C$	$-3\ln\lvert x+1\rvert - 2\ln\lvert x+2\rvert + \ln\lvert x+3\rvert + C$
(C)	(D)
$3\ln\lvert x+1\rvert - 2\ln\lvert x+2\rvert + \ln\lvert x+3\rvert + C$	$-3\ln\lvert x+1\rvert - 2\ln\lvert x+2\rvert - \ln\lvert x+3\rvert + C$

Q34.

If f is continuous on $[2,5) \cup (5,\infty)$ and discontinuous at 5, then $\int_2^\infty f(x)dx =$

(A) $\lim_{a\to 5^-} \int_2^a f(x)dx + \lim_{a\to 5^+} \int_a^7 f(x)dx + \lim_{b\to\infty} \int_7^9 bf(x)dx$
(B) $\lim_{a\to 5^-} \int_2^a f(x)dx + \lim_{a\to 5^+} \int_a^7 f(x)dx + \lim_{b\to\infty} \int_7^b f(x)dx$
(C) $\lim_{a\to 5^+} \int_2^a f(x)dx + \lim_{a\to 5^-} \int_a^7 f(x)dx + \lim_{b\to\infty} \int_7^9 bf(x)dx$
(D) $\lim_{a\to 5^+} \int_2^a f(x)dx + \lim_{a\to 5^-} \int_a^7 f(x)dx + \lim_{b\to\infty} \int_7^b f(x)dx$

Q35.

The integral $\int_0^\infty \frac{x}{x^3+1}\, dx$ is

(A)	(B)	(C)	(D)
convergent	convergent and divergent	divergent	neither convergent nor divergent

السؤال رقم 36 هو تكرار للسؤال رقم 35 و يجب أن تجيب عليه للحصول على درجته

Q36.

The integral $\int_0^\infty \frac{x}{x^3+1}\, dx$ is

(A)	(B)	(C)	(D)
convergent	convergent and divergent	divergent	neither convergent nor divergent

Q37.

The length of the curve $y = \ln(\sec x);\ 0 \le x \le \frac{\pi}{4}$, is

(A)	(B)	(C)	(D)	(E)
$\frac{\sqrt{2}+1}{\pi}$	$\ln(\sqrt{2})$	$\ln(\sqrt{2}+2)$	$\ln(\sqrt{2}+1)$	$(\sqrt{2}+1)\pi$

السؤال رقم 38 هو تكرار للسؤال رقم 37 و يجب أن تجيب عليه للحصول على درجته

Q38.

The length of the curve $y = \ln(\sec x);\ 0 \le x \le \frac{\pi}{4}$, is

(A)	(B)	(C)	(D)	(E)
$\frac{\sqrt{2}+1}{\pi}$	$\ln(\sqrt{2})$	$\ln(\sqrt{2}+2)$	$\ln(\sqrt{2}+1)$	$(\sqrt{2}+1)\pi$

C

Q39.

The curve $x = \sqrt{36 - y^2}$, where $0 \leq y \leq 3$, is rotated about the y-axis. The area of the resulting surface is

(A)	(B)	(C)	(D)	(E)
22π	46π	36π	14π	16π

السؤال رقم 40 هو تكرار للسؤال رقم 39 و يجب أن تجيب عليه للحصول على درجته

Q40.

The curve $x = \sqrt{36 - y^2}$, where $0 \leq y \leq 3$, is rotated about the y-axis. The area of the resulting surface is

(A)	(B)	(C)	(D)	(E)
22π	46π	36π	14π	16π