جامعة الملك عبدالعزيز.

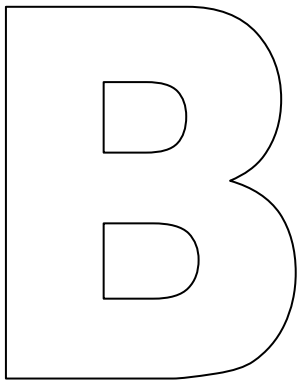
كلية العلوم. الاسم:_____

قسم الرياضيات. الرقم الجامعي:_____

math 202.
Calculus 2.

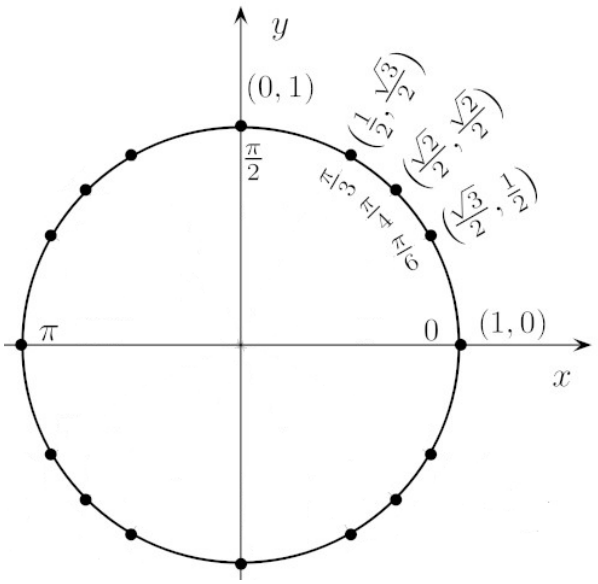
Final Exam

Date: Monday 8 / 2 / 1433 H.
Time: from 08:00 to 10:00.

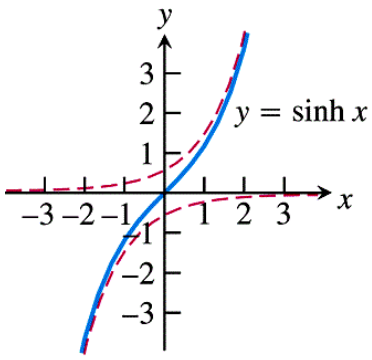
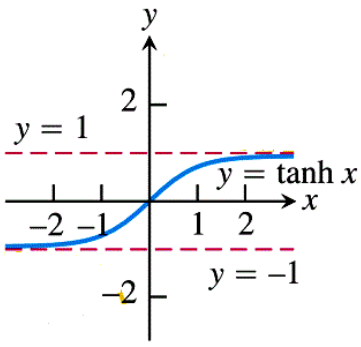


- تأكد من أن رمز نموذج الإجابة لديك هو B .
- أكتب اسمك على هذا النموذج ثم تأكد من تعبئة جميع بيانات نموذج الإجابة **خاصة رقمك الجامعي و بقلم الرصاص.**
- تأكد من تعبئة نموذج الحضور بصورة صحيحة.
- أجب عن جميع الأسئلة الآتية بتظليل الخيار الصحيح في نموذج الإجابة **بقلم الرصاص**.
- ممنوع استخدام الآلة الحاسبة.

هذه الصفحة تتضمن بعض القوانين التي قد تحتاجها لحل بعض أسئلة هذا الامتحان.



The Unit Circle

$\cosh^2 x - \sinh^2 x = 1$	$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
$\sinh(a+b) = \sinh a \cosh b + \cosh a \sinh b$	$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$
$\cosh(a+b) = \cosh a \cosh b + \sinh a \sinh b$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\displaystyle\int u \sin^{-1} u \; du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$	$\displaystyle\int u \cos^{-1} u \; du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$
	

Q1. $\lim_{x\to\infty} \sinh x =$			
(A) 1	(B) -1	(C) ∞	(D) $-\infty$

Q2. $\tanh^2 x =$			
(A) $1 - \operatorname{sech} x$	(B) $1 - \operatorname{sech}^2 x$	(C) $\operatorname{sech}^2 x \ - 1$	(D) $\operatorname{sech} x \ - 1$

Q3. If $y = \frac{\tanh^{-1}\sqrt{x}}{2}$, then $y' = \frac{dy}{dx} =$			
(A) $\frac{1}{4\sqrt{x}\ (1-x)}$	(B) $\frac{4}{3\sqrt{x}\ (1-x)}$	(C) $\frac{1}{4\sqrt{x}\ (1+x)}$	(D) $\frac{1}{\sqrt{x}\ (1-x)}$

Q4. $\int \frac{dx}{4\sqrt{x} - 4\sqrt{x^3}} =$ Hint: see question 3 above			
(A) $\frac{\tanh^{-1}\sqrt{x}}{\sqrt{2}} + C$	(B) $\frac{\tanh^{-1}\sqrt{x}}{4} + C$	(C) $\frac{4\tanh^{-1}\sqrt{x}}{3} + C$	(D) $\frac{\tanh^{-1}\sqrt{x}}{2} + C$

Q5. $\frac{d}{dx}\left(\frac{\cosh(x^2)}{2}\right) =$			
(A) $\frac{x\ \sinh(x^2)}{2}$	(B) $\frac{-x\ \sinh(x^2)}{2}$	(C) $x\ \sinh(x^2)$	(D) $-x\ \sinh(x^2)$

Q6. If $f'(x) = e^x + \frac{1}{\sqrt{1+x^2}}$ and $f(0) = -7$, then $f(x) =$			
(A) $e^x + \cosh^{-1} x + 5$	(B) $e^x + \cosh^{-1} x + 8$	(C) $e^x + \sinh^{-1} x - 8$	(D) $e^x + \sinh^{-1} x - 5$

Q7. If $\sum_{i=1}^{n} a_i = 7$ and $\sum_{i=1}^{n} b_i = -13$, then $\sum_{i=1}^{n}(3a_i + b_i) =$			
(A) 8	(B) 40	(C) 27	(D) -27

Q8.

The integral expression of $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos x_i}{x_i}\Delta x$ over the interval $[\,\pi\,,\,2\pi\,]$ is

(A)	(B)	(C)	(D)
$\int_{\pi}^{2\pi}\frac{-\sin x}{x}dx$	$\int_{\pi}^{2\pi}\frac{-\cos x}{x}dx$	$\int_{\pi}^{2\pi}\frac{\sin x}{x}dx$	$\int_{\pi}^{2\pi}\frac{\cos x}{x}dx$

Q9.

If $\int_4^9 f(x)dx=-11$ and $\int_7^9 f(x)dx=3$, then $\int_4^7\frac{f(x)}{7}dx=$

(A)	(B)	(C)	(D)
-2	-14	-7	$-\frac{11}{7}$

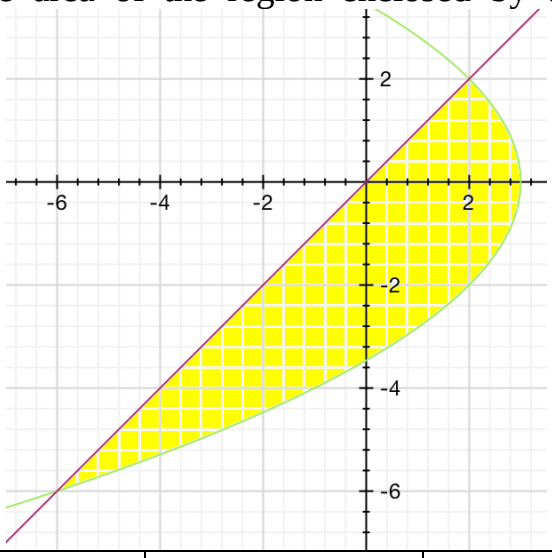
Q10.

$\frac{d}{dx}\left(\int_1^{x^3}\cos t\ dt\right)=$

(A)	(B)	(C)	(D)
$3x^2\cos(x^3)$	$-3x^2\cos(x^3)$	$3x^2\sin(x^3)$	$-3x^2\sin(x^3)$

Q11.

The area of the region enclosed by the parabolas $4x+y^2=12$ and $x=y$ is



(A)	(B)	(C)	(D)	(E)
$\frac{63}{3}$	$\frac{64}{3}$	$\frac{65}{3}$	$\frac{67}{3}$	$\frac{68}{3}$

السؤال رقم 12 هو تكرار للسؤال رقم 11 و يجب أن تجيب عليه للحصول على درجته

Q12.

The area of the region enclosed by the parabolas $4x+y^2=12$ and $x=y$ is

(A)	(B)	(C)	(D)	(E)
$\frac{63}{3}$	$\frac{64}{3}$	$\frac{65}{3}$	$\frac{67}{3}$	$\frac{68}{3}$

Q13.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{6}{5}\right)}}$ over the interval $[1, \infty)$ is

(A)	(B)	(C)	(D)	(E)
2	3	4	5	6

السؤال رقم 14 هو تكرار للسؤال رقم 13 و يجب أن تجيب عليه للحصول على درجته

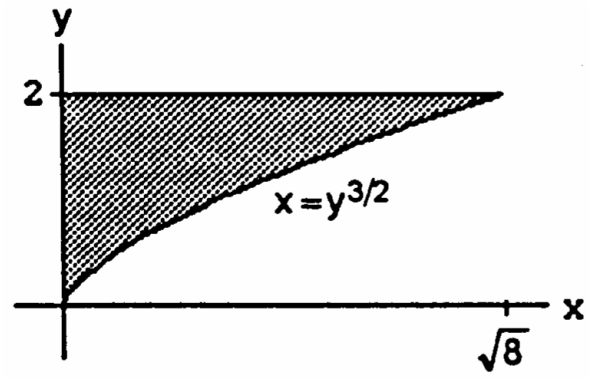
Q14.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{6}{5}\right)}}$ over the interval $[1, \infty)$ is

(A)	(B)	(C)	(D)	(E)
2	3	4	5	6

Q15.

The volume of solid generated by rotating the region bounded by curve $x = y^{\frac{3}{2}}$ and the lines $x = 0$, and $y = 2$, about the y-axis is



(A)	(B)	(C)	(D)	(E)
4π	3π	2π	π	$\frac{3}{2}\pi$

السؤال رقم 16 هو تكرار للسؤال رقم 15 و يجب أن تجيب عليه للحصول على درجته

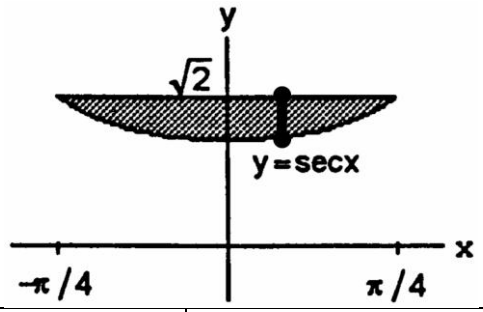
Q16.

The volume of solid generated by rotating the region bounded by curve $x = y^{\frac{3}{2}}$ and the lines $x = 0$, and $y = 2$, about the y-axis is

(A)	(B)	(C)	(D)	(E)
4π	3π	2π	π	$\frac{3}{2}\pi$

Q17.

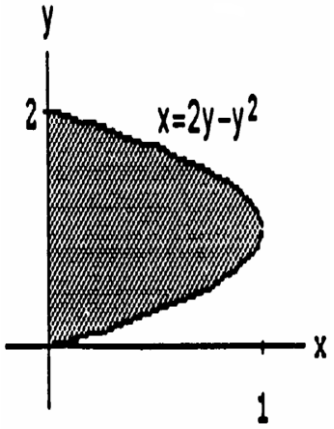
The integral which gives the volume of the solid generated by rotating about the x-axis the region bounded by the curve $y = \sec x$ where $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ and the line $y = \sqrt{2}$ is



(A) $V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec^2 x - 2)\, dx$	(B) $V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec^2 x - 4)\, dx$
(C) $V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2 - \sec^2 x)\, dx$	(D) $V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (4 - \sec^2 x)\, dx$

Q18.

By using the Shell Method, the integral which gives the volume of the solid generated by rotating about the x-axis the region bounded by the curve $x = 2y - y^2$ and the line $x = 0$ is



(A) $V = \int_0^2 2\pi\, y\, [y^2 - 2y]\, dy$	(B) $V = \int_0^2 2\pi\, y\, [2y - y^2]\, dy$
(C) $V = \int_0^1 2\pi\, y\, [y^2 - 2y]\, dy$	(D) $V = \int_0^1 2\pi\, y\, [2y - y^2]\, dy$

Q19.

$\int_0^5 \frac{dx}{x-1} =$

(A)	(B)	(C)	(D)	(E)
$\ln 4$	$\ln 3$	divergent	$\ln \frac{4}{3}$	$\ln 2$

Q20. $\int_0^1 \log_5 x \ dx =$				
(A) $\ln 5$	(B) 5	(C) divergent	(D) $\frac{1}{\ln 5}$	(E) $-\frac{1}{\ln 5}$

Q21. $\int \frac{dx}{\sqrt{12+4x-x^2}} =$ Hint: complete the square then use a suitable formula.				
(A) $\frac{\ln(4-2x)}{\sqrt{12+4x-x^2}} + C$	(B) $\frac{4-2x}{\sqrt{12+4x-x^2}} + C$	(C) $\sin^{-1}\left(\frac{x-4}{2}\right) + C$	(D) $\sin^{-1}\left(\frac{x-2}{4}\right) + C$	(E) $\tan^{-1}\left(\frac{x-4}{2}\right) + C$

السؤال رقم 22 هو تكرار للسؤال رقم 21 و يجب أن تجيب عليه للحصول على درجته

Q22. $\int \frac{dx}{\sqrt{12+4x-x^2}} =$				
(A) $\frac{\ln(4-2x)}{\sqrt{12+4x-x^2}} + C$	(B) $\frac{4-2x}{\sqrt{12+4x-x^2}} + C$	(C) $\sin^{-1}\left(\frac{x-4}{2}\right) + C$	(D) $\sin^{-1}\left(\frac{x-2}{4}\right) + C$	(E) $\tan^{-1}\left(\frac{x-4}{2}\right) + C$

Q23. $\int x^4 \ln x \ dx =$				
(A) $\frac{x^5 \ln x}{25} - \frac{x^5}{5} + C$	(B) $\frac{x^5 \ln x}{5} - \frac{x^5}{3} + C$	(C) $\frac{x^4 \ln x}{5} - \frac{x^5}{25} + C$	(D) $\frac{x^4 \ln x}{5} - \frac{x^5}{3} + C$	(E) $\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$

السؤال رقم 24 هو تكرار للسؤال رقم 23 و يجب أن تجيب عليه للحصول على درجته

Q24. $\int x^4 \ln x \ dx =$				
(A) $\frac{x^5 \ln x}{25} - \frac{x^5}{5} + C$	(B) $\frac{x^5 \ln x}{5} - \frac{x^5}{3} + C$	(C) $\frac{x^4 \ln x}{5} - \frac{x^5}{25} + C$	(D) $\frac{x^4 \ln x}{5} - \frac{x^5}{3} + C$	(E) $\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$

Q25. $\int \cos^2 x \tan^3 x \ dx =$		
(A) $\frac{1}{2}\cos^2 x - \ln\lvert\cos x\rvert + C$	(B) $\frac{1}{2}\cos^2 x + \ln\lvert\cos x\rvert + C$	(C) $-\frac{1}{2}\cos^2 x - \ln\lvert\cos x\rvert + C$
(D) $\ln\lvert\cos x\rvert - \frac{1}{2}\cos^2 x + C$	(E) $\frac{1}{2}\left(\cos^2 x - \ln\lvert\cos x\rvert\right) + C$	

السؤال رقم 26 هو تكرار للسؤال رقم 25 و يجب أن تجيب عليه للحصول على درجته

Q26. $\int \cos^2 x \tan^3 x \, dx =$		
(A) $\frac{1}{2}\cos^2 x - \ln\lvert\cos x\rvert + C$	(B) $\frac{1}{2}\cos^2 x + \ln\lvert\cos x\rvert + C$	(C) $-\frac{1}{2}\cos^2 x - \ln\lvert\cos x\rvert + C$
(D) $\ln\lvert\cos x\rvert - \frac{1}{2}\cos^2 x + C$	(E) $\frac{1}{2}\left(\cos^2 x - \ln\lvert\cos x\rvert\right) + C$	

Q27. $\int \sqrt{1-9x^2}\, dx =$ Hint: let $3x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$		
(A) $\frac{1}{6}\left[\sin^{-1}(9x^2) + 3x\sqrt{1-9x^2}\right] + C$	(B) $\frac{1}{6}\left[\sin^{-1}(3x) + x\sqrt{1-9x^2}\right] + C$	(C) $\frac{1}{6}\left[\sin^{-1}(3x) + 3x\sqrt{1-9x^2}\right] + C$
(D) $\frac{1}{6}\left[\sin^{-1}(3x) + 3\sqrt{1-9x^2}\right] + C$	(E) $\frac{1}{6}\left[\sin^{-1}(9x^2) + x\sqrt{1-9x^2}\right] + C$	

السؤال رقم 28 هو تكرار للسؤال رقم 27 و يجب أن تجيب عليه للحصول على درجته

Q28. $\int \sqrt{1-9x^2}\, dx =$ Hint: let $3x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$		
(A) $\frac{1}{6}\left[\sin^{-1}(9x^2) + 3x\sqrt{1-9x^2}\right] + C$	(B) $\frac{1}{6}\left[\sin^{-1}(3x) + x\sqrt{1-9x^2}\right] + C$	(C) $\frac{1}{6}\left[\sin^{-1}(3x) + 3x\sqrt{1-9x^2}\right] + C$
(D) $\frac{1}{6}\left[\sin^{-1}(3x) + 3\sqrt{1-9x^2}\right] + C$	(E) $\frac{1}{6}\left[\sin^{-1}(9x^2) + x\sqrt{1-9x^2}\right] + C$	

Q29. Using the substitution way with $u = \tan(2\theta)$, the evaluation of the integral $\int 4\sec^2(2\theta)\tan(2\theta)\, d\theta$ is			
(A) $\tan^2(2\theta) + C$	(B) $-\tan^2(2\theta) + C$	(C) $\cot^2(2\theta) + C$	(D) $-\cot^2(2\theta) + C$

Q30. Using the substitution way with $u = \sec(2\theta)$, the evaluation of the integral $\int 4\sec^2(2\theta)\tan(2\theta)\, d\theta$ is			
(A) $-\csc^2(2\theta) + C$	(B) $\csc^2(2\theta) + C$	(C) $-\sec^2(2\theta) + C$	(D) $\sec^2(2\theta) + C$

Q31.

$\int 2\cos^{-1}\sqrt{x}\ dx =$ Hint: use a suitable formula.

(A) $(2x-1)\cos^{-1}\sqrt{x} - x\sqrt{1-x} + C$	(B) $(2x-1)\cos^{-1}\sqrt{x} - \sqrt{x-x^2} + C$
(C) $\left(2\sqrt{x}-1\right)\cos^{-1}\sqrt{x} - x\sqrt{1-x} + C$	(D) $\left(2\sqrt{x}-1\right)\cos^{-1}\sqrt{x} - \sqrt{x-x^2} + C$

Q32.

If $\frac{9x^2+14x-28}{x\,(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$, then

(A) $A=-7, B=-1, C=3$	(B) $A=-7, B=-1, C=-3$	(C) $A=7, B=-1, C=3$	(D) $A=7, B=1, C=3$

Q33.

$\int \frac{9x^2+14x-28}{x\,(x-1)(x+4)}dx =$

(A) $7\ln\lvert x\rvert - \ln\lvert x-1\rvert + 3\ln\lvert x+4\rvert + C$	(B) $-7\ln\lvert x\rvert - \ln\lvert x-1\rvert - 3\ln\lvert x+4\rvert + C$
(C) $-7\ln\lvert x\rvert - \ln\lvert x-1\rvert + 3\ln\lvert x+4\rvert + C$	(D) $7\ln\lvert x\rvert + \ln\lvert x-1\rvert + 3\ln\lvert x+4\rvert + C$

Q34.

If f is continuous on $(-\infty, 1) \cup (1, 2]$ and discontinuous at 1, then $\int_{-\infty}^{2} f(x)dx =$

(A) $\lim_{c\to-\infty}\int_{-4}^{0} cf(x)dx + \lim_{b\to1^+}\int_{0}^{b} f(x)dx + \lim_{b\to1^-}\int_{b}^{2} f(x)dx$

(B) $\lim_{c\to-\infty}\int_{c}^{0} f(x)dx + \lim_{b\to1^+}\int_{0}^{b} f(x)dx + \lim_{b\to1^-}\int_{b}^{2} f(x)dx$

(C) $\lim_{c\to-\infty}\int_{-4}^{0} cf(x)dx + \lim_{b\to1^-}\int_{0}^{b} f(x)dx + \lim_{b\to1^+}\int_{b}^{2} f(x)dx$

(D) $\lim_{c\to-\infty}\int_{c}^{0} f(x)dx + \lim_{b\to1^-}\int_{0}^{b} f(x)dx + \lim_{b\to1^+}\int_{b}^{2} f(x)dx$

Q35.

The integral $\int_{1}^{\infty} \frac{2+e^{-x}}{x}\ dx$ is

(A) neither convergent nor divergent	(B) divergent	(C) convergent and divergent	(D) convergent

السؤال رقم 36 هو تكرار للسؤال رقم 35 و يجب أن تجيب عليه للحصول على درجته

Q36.

The integral $\int_{1}^{\infty} \frac{2+e^{-x}}{x}\ dx$ is

(A) neither convergent nor divergent	(B) divergent	(C) convergent and divergent	(D) convergent

Q37. The length of the curve $y = \ln(\cos x);\ 0 \le x \le \frac{\pi}{3}$, is				
(A) $(2+\sqrt{3})\pi$	(B) $\ln(1+\sqrt{3})$	(C) $\ln(2+\sqrt{3})$	(D) $\frac{2+\sqrt{3}}{\pi}$	(E) $\ln(3+\sqrt{2})$

السؤال رقم 38 هو تكرار للسؤال رقم 37 و يجب أن تجيب عليه للحصول على درجته

Q38. The length of the curve $y = \ln(\cos x);\ 0 \le x \le \frac{\pi}{3}$, is				
(A) $(2+\sqrt{3})\pi$	(B) $\ln(1+\sqrt{3})$	(C) $\ln(2+\sqrt{3})$	(D) $\frac{2+\sqrt{3}}{\pi}$	(E) $\ln(3+\sqrt{2})$

Q39. The curve $x = \sqrt{16 - y^2}$, where $0 \le y \le 2$, is rotated about the y-axis. The area of the resulting surface is				
(A) 75π	(B) 42π	(C) 2π	(D) 14π	(E) 16π

السؤال رقم 40 هو تكرار للسؤال رقم 39 و يجب أن تجيب عليه للحصول على درجته

Q40. The curve $x = \sqrt{16 - y^2}$, where $0 \le y \le 2$, is rotated about the y-axis. The area of the resulting surface is				
(A) 75π	(B) 42π	(C) 2π	(D) 14π	(E) 16π