جامعة الملك عبدالعزيز.

كلية العلوم. الاسم:______________________________

قسم الرياضيات. الرقم الجامعي:________________________

math 202.
Calculus 2.

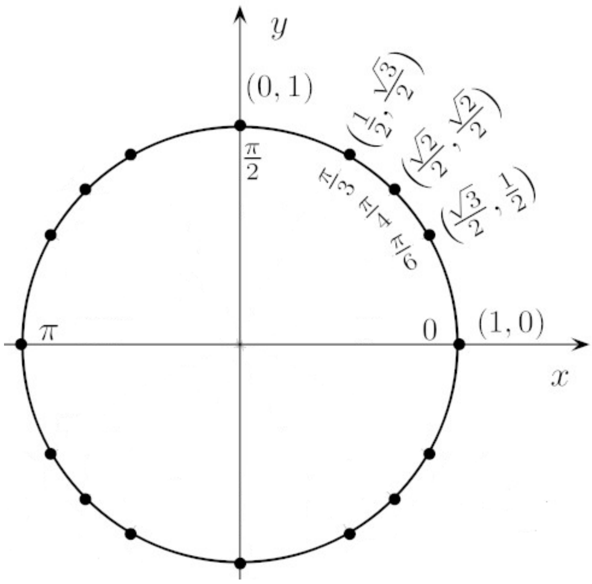
Final Exam

Date: Monday 8 / 2 / 1433 H.
Time: from 08:00 to 10:00.

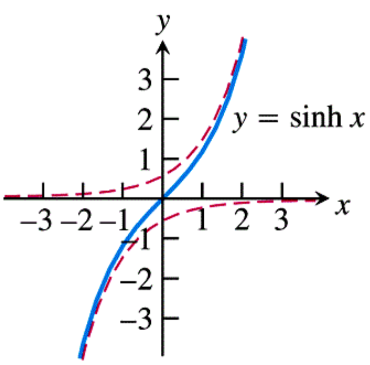
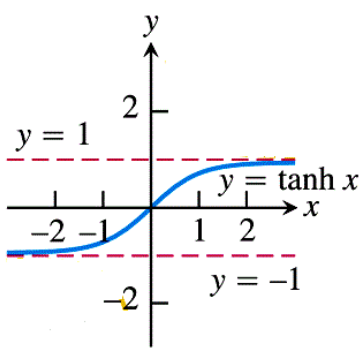
# A

- تأكد من أن رمز نموذج الإجابة لديك هو A .
- أكتب اسمك على هذا النموذج ثم تأكد من تعبئة جميع بيانات نموذج الإجابة **خاصة رقمك الجامعي و بقلم الرصاص.**
- تأكد من تعبئة نموذج الحضور بصورة صحيحة.
- أجب عن جميع الأسئلة الآتية بتظليل الخيار الصحيح في نموذج الإجابة **بقلم الرصاص**.
- ممنوع استخدام الآلة الحاسبة.

هذه الصفحة تتضمن بعض القوانين التي قد تحتاجها لحل بعض أسئلة هذا الامتحان.




The Unit Circle

| | |
|---|---|
| $\cosh^2 x - \sinh^2 x = 1$ | $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ |
| $\sinh(a+b) = \sinh a \cosh b + \cosh a \sinh b$ | $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ |
| $\cosh(a+b) = \cosh a \cosh b + \sinh a \sinh b$ | $\sin 2\theta = 2\sin\theta\cos\theta$ |
| $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$ | $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + C$ |
| $\int u\sin^{-1}u\ du = \dfrac{2u^2-1}{4}\sin^{-1}u + \dfrac{u\sqrt{1-u^2}}{4} + C$ | $\int u\cos^{-1}u\ du = \dfrac{2u^2-1}{4}\cos^{-1}u - \dfrac{u\sqrt{1-u^2}}{4} + C$ |
|   |   |

| Q1. $\lim_{x\to\infty} \tanh x =$ | | | |
|---|---|---|---|
| (A) $-1$ | (B) $1$ | (C) $-\infty$ | (D) $\infty$ |

| Q2. $\sinh 2x =$ | | | |
|---|---|---|---|
| (A) $1+\operatorname{csch}^2 x$ | (B) $1-\operatorname{sech}^2 x$ | (C) $\sinh x \ \cosh x$ | (D) $2\sinh x \ \cosh x$ |

| Q3. If $y = \tanh^{-1}\sqrt{x}$ , then $y' = \frac{dy}{dx} =$ | | | |
|---|---|---|---|
| (A) $\frac{3}{2\sqrt{x}\ \ (1-x)}$ | (B) $\frac{1}{\sqrt{x}\ \ (1-x)}$ | (C) $\frac{1}{2\sqrt{x}\ \ (1-x)}$ | (D) $\frac{1}{2\sqrt{x}\ \ (1+x)}$ |

| Q4. $\int \frac{dx}{2\sqrt{x} - 2\sqrt{x^3}} =$ Hint: see question 3 above | | | |
|---|---|---|---|
| (A) $\tanh^{-1}\sqrt{x} + C$ | (B) $2\tanh^{-1}\sqrt{x} + C$ | (C) $4\tanh^{-1}\sqrt{x} + C$ | (D) $\frac{\tanh^{-1}\sqrt{x}}{2} + C$ |

| Q5. $\frac{d}{dx}(\cosh(x^2)) =$ | | | |
|---|---|---|---|
| (A) $2x\ \sinh(x^2)$ | (B) $-2x\ \sinh(x^2)$ | (C) $2\ \sinh(x^2)$ | (D) $-2\ \sinh(x^2)$ |

| Q6. If $f'(x) = e^x + \frac{5}{\sqrt{1+x^2}}$ and $f(0) = 4,$ then $f(x) =$ | | | |
|---|---|---|---|
| (A) $e^x + 5\cosh^{-1} x + 5$ | (B) $e^x + 5\cosh^{-1} x + 3$ | (C) $e^x + 5\sinh^{-1} x + 5$ | (D) $e^x + 5\sinh^{-1} x + 3$ |

| Q7. If $\sum_{i=1}^{n} a_i = 7$ and $\sum_{i=1}^{n} b_i = -13$ , then $\sum_{i=1}^{n} (2a_i - b_i) =$ | | | |
|---|---|---|---|
| (A) $8$ | (B) $40$ | (C) $27$ | (D) $-27$ |

A

Q8.

The integral expression of $\lim_{n\to\infty}\sum_{i=1}^{n} x_i \ln\left(1+x_i^2\right)\Delta x$ over the interval $[\,2\,,6\,]$ is

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| $\int_2^6 x(1+x^2)\,dx$ | $\int_2^6 x\ln(x^2)\,dx$ | $\int_2^6 x\ln(1+x^2)\,dx$ | $\int_2^6 \frac{x}{1+x^2}\,dx$ |

Q9.

If $\int_1^7 f(x)dx = 18$ and $\int_4^7 f(x)dx = -2$, then $\int_1^4 \frac{f(x)}{5}\,dx =$

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| 20 | 4 | 5 | $\frac{18}{5}$ |

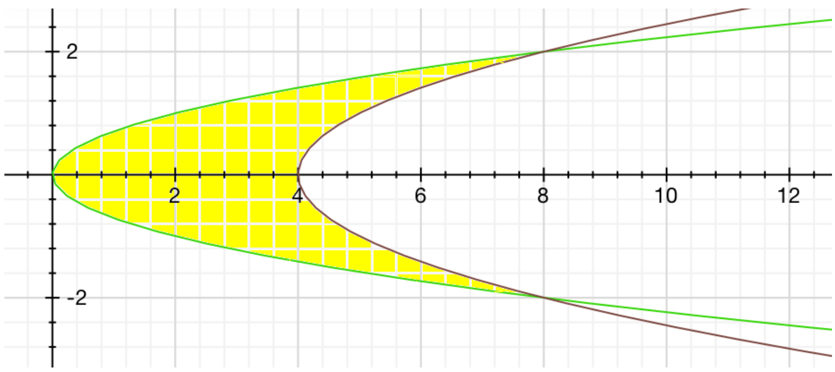
Q10.

$\frac{d}{dx}\left(\int_1^{x^4+1} \sec t\ dt\right) =$

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| $4x^3\sec(x^4)$ | $4x^3\sec(x^4+1)$ | $\sec(x^4+1)\tan(x^4+1)$ | $\frac{\sec(x^4+1)\tan(x^4+1)}{4x^3}$ |

Q11.

The area of the region enclosed by the parabolas $x = 2y^2$ and $x = 4 + y^2$ is




| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{32}{3}$ | $\frac{31}{3}$ | $\frac{29}{3}$ | $\frac{28}{3}$ | $\frac{26}{3}$ |

السؤال رقم 12 هو تكرار للسؤال رقم 11 و يجب أن تجيب عليه للحصول على درجته

Q12.

The area of the region enclosed by the parabolas $x = 2y^2$ and $x = 4 + y^2$ is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{32}{3}$ | $\frac{31}{3}$ | $\frac{29}{3}$ | $\frac{28}{3}$ | $\frac{26}{3}$ |

Q13.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{7}{6}\right)}}$ over the interval $[1, \infty)$ is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| 2 | 3 | 4 | 5 | 6 |

السؤال رقم 14 هـو تكرار للسؤال رقم 13 و يجب أن تجيب عليه للحصول على درجته

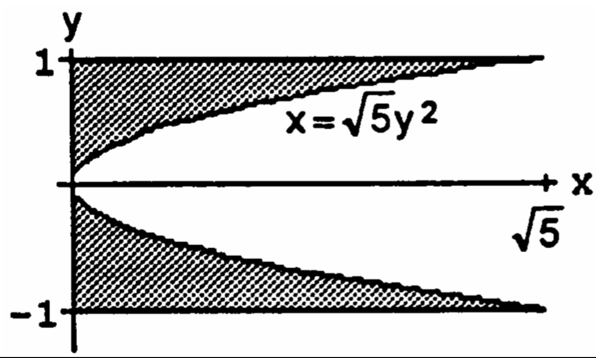
Q14.

The area of the region below the graph of $y = \frac{1}{x^{\left(\frac{7}{6}\right)}}$ over the interval $[1, \infty)$ is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| 2 | 3 | 4 | 5 | 6 |

Q15.

The volume of solid generated by rotating the region bounded by curve $x = \sqrt{5}\ \ y^2$ and the lines $x = 0$, $y = -1$, and $y = 1$, about the $y$-axis is




| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ |

السؤال رقم 16 هـو تكرار للسؤال رقم 15 و يجب أن تجيب عليه للحصول على درجته

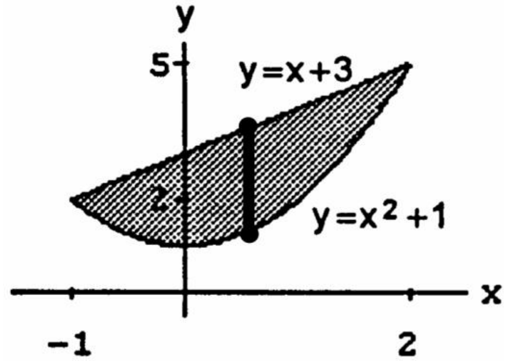
Q16.

The volume of solid generated by rotating the region bounded by curve $x = \sqrt{5}\ \ y^2$ and the lines $x = 0$, $y = -1$, and $y = 1$, about the $y$-axis is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ |

Q17.

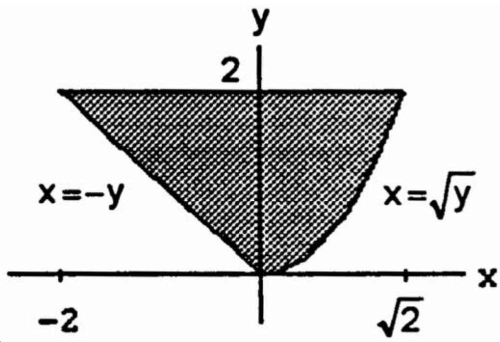
The integral which gives the volume of the solid generated by rotating about the $x$-axis the region bounded by the curve $y = x^2 + 1$ and the line $y = x + 3$ is




| | |
|---|---|
| (A) $V=\pi\int_{-1}^{2}[(x^2+1)^2-(x+3)^2]\,dx$ | (B) $V=\pi\int_{-1}^{2}[(x+3)^2-(x^2+1)^2]\,dx$ |
| (C) $V=\pi\int_{0}^{5}[(x^2+1)^2-(x+3)^2]\,dx$ | (D) $V=\pi\int_{0}^{5}[(x+3)^2-(x^2+1)^2]\,dx$ |

Q18.

By using the Shell Method, the integral which gives the volume of the solid generated by rotating about the $x$-axis the region bounded by the curve $x=\sqrt{y}$ and the lines $x=-y$ and $y=2$ is




| | |
|---|---|
| (A) $V=\int_{0}^{2}2\pi y\left[\sqrt{y}-(-y)\right]dy$ | (B) $V=\int_{0}^{2}2\pi y\left[\sqrt{y}+(-y)\right]dy$ |
| (C) $V=\int_{-2}^{\sqrt{2}}2\pi y\left[\sqrt{y}-(-y)\right]dy$ | (D) $V=\int_{-2}^{\sqrt{2}}2\pi y\left[\sqrt{y}+(-y)\right]dy$ |

Q19.

$$\int_0^6 \frac{dx}{x-1}=$$

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\ln 5$ | $\ln\frac{6}{5}$ | $\ln 2$ | divergent | $\ln 3$ |

A

| Q20. $\int_0^1 \log_3 x \; dx =$ | | | | |
|---|---|---|---|---|
| (A) $\frac{1}{\ln 3}$ | (B) $-\frac{1}{\ln 3}$ | (C) divergent | (D) $\ln 3$ | (E) $3$ |

| Q21. $\int \frac{dx}{\sqrt{3+2x-x^2}} =$ | | Hint: complete the square then use a suitable formula. | | |
|---|---|---|---|---|
| (A) $\frac{\ln(2-2x)}{\sqrt{3+2x-x^2}} + C$ | (B) $\frac{2-2x}{\sqrt{3+2x-x^2}} + C$ | (C) $\sin^{-1}\left(\frac{x-2}{2}\right) + C$ | (D) $\tan^{-1}\left(\frac{x-2}{2}\right) + C$ | (E) $\sin^{-1}\left(\frac{x-1}{2}\right) + C$ |

السؤال رقم 22 هو تكرار للسؤال رقم 21 و يجب أن تجيب عليه للحصول على درجته

| Q22. $\int \frac{dx}{\sqrt{3+2x-x^2}} =$ | | | | |
|---|---|---|---|---|
| (A) $\frac{\ln(2-2x)}{\sqrt{3+2x-x^2}} + C$ | (B) $\frac{2-2x}{\sqrt{3+2x-x^2}} + C$ | (C) $\sin^{-1}\left(\frac{x-2}{2}\right) + C$ | (D) $\tan^{-1}\left(\frac{x-2}{2}\right) + C$ | (E) $\sin^{-1}\left(\frac{x-1}{2}\right) + C$ |

| Q23. $\int x^5 \ln x \; dx =$ | | | | |
|---|---|---|---|---|
| (A) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$ | (B) $\frac{x^6 \ln x}{6} - \frac{x^6}{3} + C$ | (C) $\frac{x^5 \ln x}{6} - \frac{x^6}{36} + C$ | (D) $\frac{x^5 \ln x}{6} - \frac{x^6}{3} + C$ | (E) $\frac{x^6 \ln x}{36} - \frac{x^6}{6} + C$ |

السؤال رقم 24 هو تكرار للسؤال رقم 23 و يجب أن تجيب عليه للحصول على درجته

| Q24. $\int x^5 \ln x \; dx =$ | | | | |
|---|---|---|---|---|
| (A) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$ | (B) $\frac{x^6 \ln x}{6} - \frac{x^6}{3} + C$ | (C) $\frac{x^5 \ln x}{6} - \frac{x^6}{36} + C$ | (D) $\frac{x^5 \ln x}{6} - \frac{x^6}{3} + C$ | (E) $\frac{x^6 \ln x}{36} - \frac{x^6}{6} + C$ |

| Q25. $\int \cot^5\theta \sin^4\theta \, d\theta =$ | | |
|---|---|---|
| (A) $\ln\lvert\sin\theta\rvert + \sin^2\theta + \frac{1}{4}\sin^4\theta + C$ | (B) $\ln\lvert\sin\theta\rvert - \sin^2\theta + \frac{1}{3}\sin^3\theta + C$ | (C) $\ln\lvert\sin\theta\rvert - \sin^2\theta + \frac{1}{4}\sin^4\theta + C$ |
| (D) $\ln\lvert\sin\theta\rvert + \sin^2\theta + \frac{1}{3}\sin^3\theta + C$ | (E) $2\ln\lvert\sin\theta\rvert - \sin^2\theta + \frac{1}{3}\sin^3\theta + C$ | |

السؤال رقم 26 هو تكرار للسؤال رقم 25 و يجب أن تجيب عليه للحصول على درجته

| Q26. $\int \cot^5\theta \sin^4\theta \, d\theta =$ | | |
|---|---|---|
| (A) $\ln\lvert\sin\theta\rvert + \sin^2\theta + \frac{1}{4}\sin^4\theta + C$ | (B) $\ln\lvert\sin\theta\rvert - \sin^2\theta + \frac{1}{3}\sin^3\theta + C$ | (C) $\ln\lvert\sin\theta\rvert - \sin^2\theta + \frac{1}{4}\sin^4\theta + C$ |
| (D) $\ln\lvert\sin\theta\rvert + \sin^2\theta + \frac{1}{3}\sin^3\theta + C$ | (E) $2\ln\lvert\sin\theta\rvert - \sin^2\theta + \frac{1}{3}\sin^3\theta + C$ | |

| Q27. $\int \sqrt{1-4x^2}\, dx =$ Hint: let $2x = \sin\theta$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ | | |
|---|---|---|
| (A) $\frac{1}{4}\left[\sin^{-1}(4x^2) + 2x\sqrt{1-4x^2}\right] + C$ | (B) $\frac{1}{4}\left[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}\right] + C$ | (C) $\frac{1}{4}\left[\sin^{-1}(2x) + x\sqrt{1-4x^2}\right] + C$ |
| (D) $\frac{1}{4}\left[\sin^{-1}(2x) + 2\sqrt{1-4x^2}\right] + C$ | (E) $\frac{1}{4}\left[\sin^{-1}(4x^2) + x\sqrt{1-4x^2}\right] + C$ | |

السؤال رقم 28 هو تكرار للسؤال رقم 27 و يجب أن تجيب عليه للحصول على درجته

| Q28. $\int \sqrt{1-4x^2}\, dx =$ | | |
|---|---|---|
| (A) $\frac{1}{4}\left[\sin^{-1}(4x^2) + 2x\sqrt{1-4x^2}\right] + C$ | (B) $\frac{1}{4}\left[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}\right] + C$ | (C) $\frac{1}{4}\left[\sin^{-1}(2x) + x\sqrt{1-4x^2}\right] + C$ |
| (D) $\frac{1}{4}\left[\sin^{-1}(2x) + 2\sqrt{1-4x^2}\right] + C$ | (E) $\frac{1}{4}\left[\sin^{-1}(4x^2) + x\sqrt{1-4x^2}\right] + C$ | |

| Q29. Using the substitution way with $u = \cot(2\theta)$, the evaluation of the integral $\int \csc^2(2\theta)\cot(2\theta)\, d\theta$ is | | | |
|---|---|---|---|
| (A) $\frac{1}{4}\tan^2(2\theta) + C$ | (B) $-\frac{1}{4}\tan^2(2\theta) + C$ | (C) $\frac{1}{4}\cot^2(2\theta) + C$ | (D) $-\frac{1}{4}\cot^2(2\theta) + C$ |

| Q30. Using the substitution way with $u = \csc(2\theta)$, the evaluation of the integral $\int \csc^2(2\theta)\cot(2\theta)\, d\theta$ is | | | |
|---|---|---|---|
| (A) $-\frac{1}{4}\sec^2(2\theta) + C$ | (B) $\frac{1}{4}\sec^2(2\theta) + C$ | (C) $-\frac{1}{4}\csc^2(2\theta) + C$ | (D) $\frac{1}{4}\csc^2(2\theta) + C$ |

**Q31.**

$$\int \frac{\sin^{-1}\sqrt{x}}{2}\,dx =$$

Hint: use a suitable formula.

| | |
|---|---|
| (A) $\dfrac{(2x-1)\sin^{-1}\sqrt{x} + \sqrt{x-x^2}}{4} + C$ | (B) $\dfrac{\left(2\sqrt{x}-1\right)\sin^{-1}\sqrt{x} + \sqrt{x-x^2}}{4} + C$ |
| (C) $\dfrac{(2x-1)\sin^{-1}\sqrt{x} + x\sqrt{1-x}}{4} + C$ | (D) $\dfrac{\left(2\sqrt{x}-1\right)\sin^{-1}\sqrt{x} + x\sqrt{1-x}}{4} + C$ |

**Q32.**

If $\dfrac{9x^2-45x-120}{x\,(x-5)(x+3)} = \dfrac{A}{x} + \dfrac{B}{x-5} + \dfrac{C}{x+3}$ , then

| | | | |
|---|---|---|---|
| (A) $A=-8, B=3, C=-4$ | (B) $A=-8, B=3, C=4$ | (C) $A=8, B=3, C=4$ | (D) $A=8, B=-3, C=4$ |

**Q33.**

$$\int \frac{9x^2-45x-120}{x\,(x-5)(x+3)}\,dx =$$

| | |
|---|---|
| (A) $-8\ln\lvert x\rvert + 3\ln\lvert x-5\rvert - 4\ln\lvert x+3\rvert + C$ | (B) $8\ln\lvert x\rvert - 3\ln\lvert x-5\rvert + 4\ln\lvert x+3\rvert + C$ |
| (C) $8\ln\lvert x\rvert + 3\ln\lvert x-5\rvert + 4\ln\lvert x+3\rvert + C$ | (D) $-8\ln\lvert x\rvert + 3\ln\lvert x-5\rvert + 4\ln\lvert x+3\rvert + C$ |

**Q34.**

If $f$ is continuous on $[\,2\,,3\,)\cup(\,3\,,\infty\,)$ and discontinuous at 3, then $\int_2^{\infty} f(x)dx =$

(A) $\lim\limits_{a\to 3^-}\int_2^a f(x)dx + \lim\limits_{a\to 3^+}\int_a^4 f(x)dx + \lim\limits_{b\to\infty}\int_4^b f(x)dx$

(B) $\lim\limits_{a\to 3^-}\int_2^a f(x)dx + \lim\limits_{a\to 3^+}\int_a^4 f(x)dx + \lim\limits_{b\to\infty}\int_4^5 bf(x)dx$

(C) $\lim\limits_{a\to 3^+}\int_2^a f(x)dx + \lim\limits_{a\to 3^-}\int_a^4 f(x)dx + \lim\limits_{b\to\infty}\int_4^b f(x)dx$

(D) $\lim\limits_{a\to 3^+}\int_2^a f(x)dx + \lim\limits_{a\to 3^-}\int_a^4 f(x)dx + \lim\limits_{b\to\infty}\int_4^5 bf(x)dx$

**Q35.**

The integral $\int_0^{\infty} \frac{x}{x^3+1}\,dx$ is

| | | | |
|---|---|---|---|
| (A) divergent | (B) neither convergent nor divergent | (C) convergent | (D) convergent and divergent |

السؤال رقم 36 هو تكرار للسؤال رقم 35 و يجب أن تجيب عليه للحصول على درجته

**Q36.**

The integral $\int_0^{\infty} \frac{x}{x^3+1}\, dx$ is

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| divergent | neither convergent nor divergent | convergent | convergent and divergent |

**Q37.**

The length of the curve $y = \ln(\sec x);\ 0 \le x \le \frac{\pi}{4}$, is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $(\sqrt{2}+1)\pi$ | $\ln(\sqrt{2}+1)$ | $\ln(\sqrt{2})$ | $\frac{\sqrt{2}+1}{\pi}$ | $\ln(\sqrt{2}+2)$ |

السؤال رقم 38 هو تكرار للسؤال رقم 37 و يجب أن تجيب عليه للحصول على درجته

**Q38.**

The length of the curve $y = \ln(\sec x);\ 0 \le x \le \frac{\pi}{4}$, is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $(\sqrt{2}+1)\pi$ | $\ln(\sqrt{2}+1)$ | $\ln(\sqrt{2})$ | $\frac{\sqrt{2}+1}{\pi}$ | $\ln(\sqrt{2}+2)$ |

**Q39.**

The curve $x = \sqrt{64 - y^2}$, where $0 \le y \le 4$, is rotated about the $y$-axis. The area of the resulting surface is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $25\pi$ | $4\pi$ | $32\pi$ | $64\pi$ | $46\pi$ |

السؤال رقم 40 هو تكرار للسؤال رقم 39 و يجب أن تجيب عليه للحصول على درجته

**Q40.**

The curve $x = \sqrt{64 - y^2}$, where $0 \le y \le 4$, is rotated about the $y$-axis. The area of the resulting surface is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $25\pi$ | $4\pi$ | $32\pi$ | $64\pi$ | $46\pi$ |