جامعة الملك عبدالعزيز.

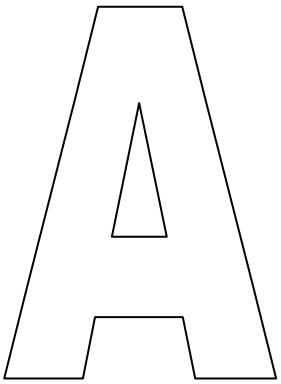
كلية العلوم. الاسم:_____

قسم الرياضيات. الرقم الجامعي:_____

math 202.
Calculus 2.

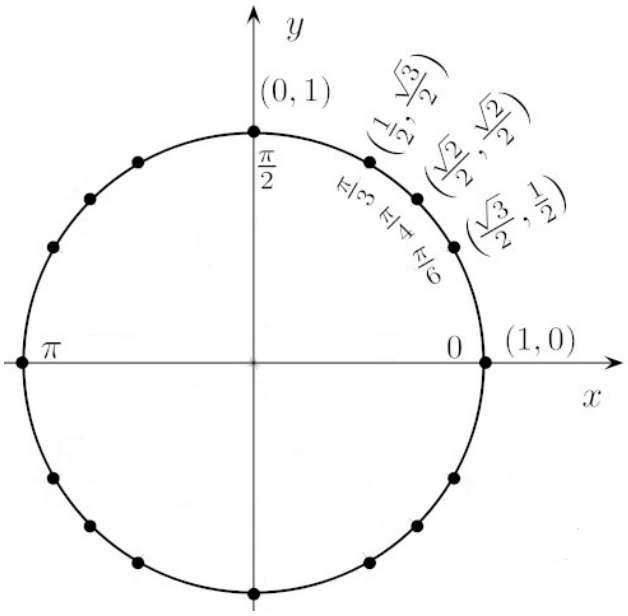
Second Exam

Date: Sunday 16 / 1 / 1433 H.
Time: from 20:15 to 21:45.




- تأكد من أن رمز نموذج الإجابة لديك هو A .
- أكتب اسمك على هذا النموذج ثم تأكد من تعبئة جميع بيانات نموذج الإجابة **خاصة رقمك الجامعي و بقلم الرصاص.**
- تأكد من تعبئة نموذج الحضور بصورة صحيحة.
- أجب عن جميع الأسئلة الآتية بتظليل الخيار الصحيح في نموذج الإجابة **بقلم الرصاص**.
- ممنوع استخدام الآلة الحاسبة.

هذه الصفحة تتضمن بعض القوانين التي قد تحتاجها لحل بعض أسئلة هذا الامتحان.



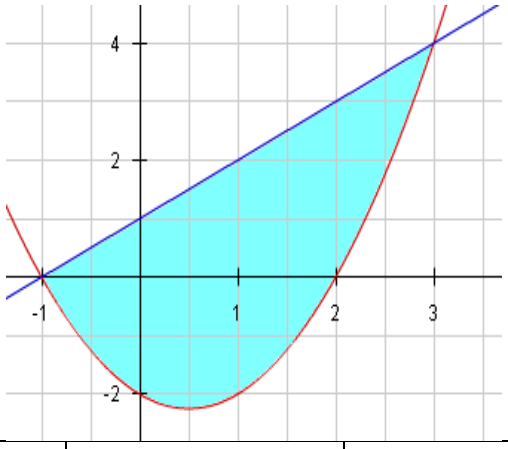

The Unit Circle

| | |
|---|---|
| $\sin mx \sin nx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$ | $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ |
| $\sin mx \cos nx = \frac{1}{2}[\sin(m-n)x + \sin(m+n)x]$ | $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ |
| $\cos mx \cos nx = \frac{1}{2}[\cos(m-n)x + \cos(m+n)x]$ | $\sin 2\theta = 2\sin\theta\cos\theta$ |
| $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$ | $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$ | $\int e^{au}\sin bu\ du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$ |
| $\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left\lvert\frac{a+bu}{u}\right\rvert + C$ | $\int u^n \ln u\ du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln(u) - 1] + C$ |
| $\frac{d}{dx}(e^x) = e^x$ | $\frac{d}{dx}(a^x) = a^x \cdot \ln a$ |

A

Q1.

The area of the region enclosed by the parabola $y = x^2 - x - 2$ and the line $y = x + 1$ is




| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{35}{3}$ | $\frac{34}{3}$ | $\frac{32}{3}$ | $\frac{31}{3}$ | $\frac{29}{3}$ |

السؤال رقم 2 هو تكرار للسؤال رقم 1 و يجب أن تجيب عليه للحصول على درجته

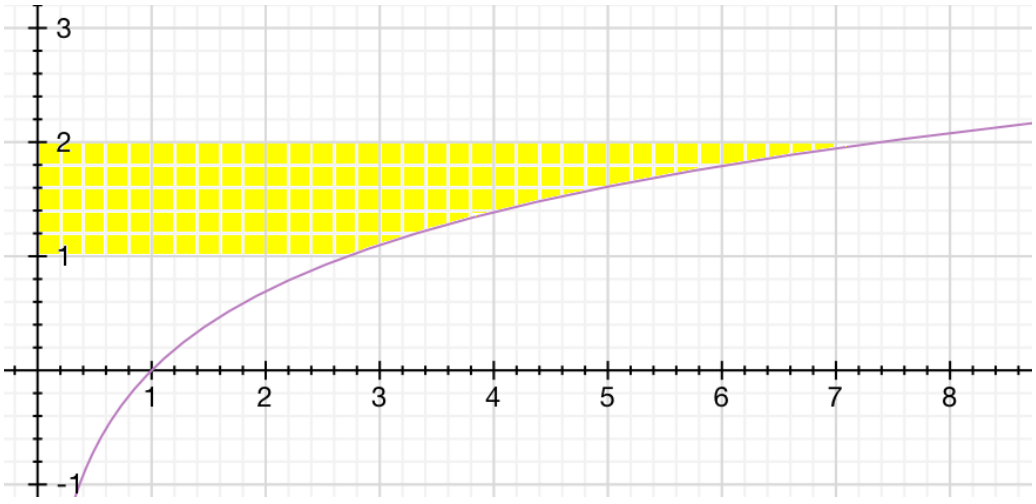
Q2.

The area of the region enclosed the parabola $y = x^2 - x - 2$ and the line $y = x + 1$ is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{35}{3}$ | $\frac{34}{3}$ | $\frac{32}{3}$ | $\frac{31}{3}$ | $\frac{29}{3}$ |

Q3.

The volume of the solid obtained by rotating the region bounded by the curve $x = e^y$ and the lines $y = 1$, $y = 2$, and $x = 0$ about the $y$-axis is




| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{\pi e^2}{2}(e^2+1)$ | $\frac{\pi e^2}{2}(e^2-2)$ | $\frac{e^2}{2}(e^2-1)$ | $\frac{\pi}{2}(e^2-1)$ | $\frac{\pi e^2}{2}(e^2-1)$ |

السؤال رقم 4 هو تكرار للسؤال رقم 3 و يجب أن تجيب عليه للحصول على درجته

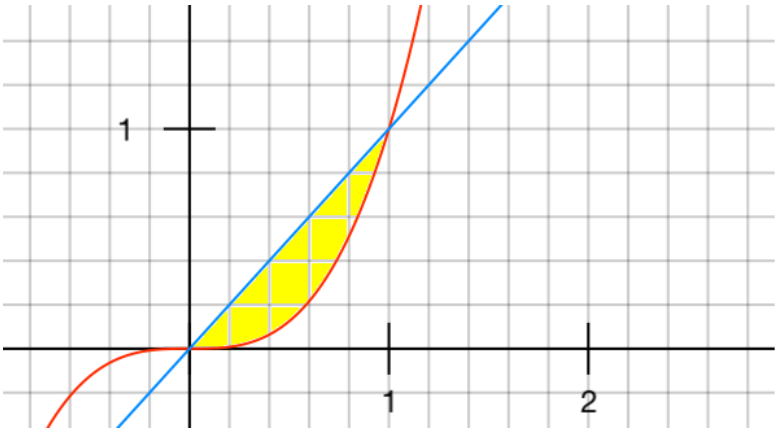
Q4.

The volume of the solid obtained by rotating the region bounded by the curve $x = e^y$ and the lines $y = 1$, $y = 2$, and $x = 0$ about the $y$-axis is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{\pi e^2}{2}(e^2+1)$ | $\frac{\pi e^2}{2}(e^2-2)$ | $\frac{e^2}{2}(e^2-1)$ | $\frac{\pi}{2}(e^2-1)$ | $\frac{\pi e^2}{2}(e^2-1)$ |

Q5.

The volume of the solid obtained by rotating the region bounded by the curve $y = x^3$ and the line $y = x$, where $x \geq 0$, about the $x$-axis is




| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{11}{21}\pi$ | $\frac{13}{21}\pi$ | $\frac{7}{21}\pi$ | $\frac{4}{21}\pi$ | $\frac{5}{21}\pi$ |

السؤال رقم 6 هو تكرار للسؤال رقم 5 و يجب أن تجيب عليه للحصول على درجته

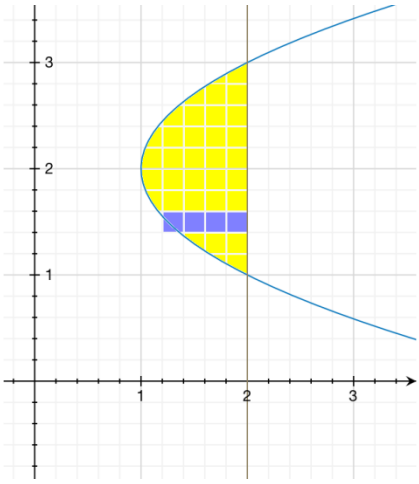
Q6.

The volume of the solid obtained by rotating the region bounded by the curve $y = x^3$ and the line $y = x$, where $x \geq 0$, about the $x$-axis is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{11}{21}\pi$ | $\frac{13}{21}\pi$ | $\frac{7}{21}\pi$ | $\frac{4}{21}\pi$ | $\frac{5}{21}\pi$ |

Q7.

The volume of the solid obtained by rotating the region bounded by the curve $x = 1 + (y-2)^2$ and the line $x = 2$ about the $x$-axis is




| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{16}{3}\pi$ | $\frac{14}{3}\pi$ | $\frac{13}{3}\pi$ | $\frac{11}{3}\pi$ | $\frac{10}{3}\pi$ |

السؤال رقم 8 هو تكرار للسؤال رقم 7 و يجب أن تجيب عليه للحصول على درجته

Q8.

The volume of the solid obtained by rotating the region bounded by the curve $x = 1 + (y-2)^2$ and the line $x = 2$ about the $x$-axis is

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{16}{3}\pi$ | $\frac{14}{3}\pi$ | $\frac{13}{3}\pi$ | $\frac{11}{3}\pi$ | $\frac{10}{3}\pi$ |

Q9.

$$\int x \cos 2x \, dx =$$

| (A) | (B) |
|---|---|
| $\frac{x \sin 2x}{4} + \frac{\cos 2x}{2} + C$ | $\frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C$ |
| (C) | (D) |
| $\frac{x \sin 2x}{2} + \frac{\cos 2x}{2} + C$ | $\frac{x \sin 2x}{4} + \frac{\cos 2x}{4} + C$ |

Q10.

$$\int \ln(2x) \, dx =$$

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| $x\ln(2x) - 2x + C$ | $x\ln(2x) + x + C$ | $\frac{1}{2}x\ln(2x) - x + C$ | $x\ln(2x) - x + C$ |

| Q11. $\int \sin^2 x \ \cos^3 x \, dx =$ | |
|---|---|
| (A) $3\sin^3 x - \frac{1}{5} \sin^5 x + C$ | (B) $\frac{1}{3}\sin^3 x - \frac{1}{5} \sin^5 x + C$ |
| (C) $\frac{1}{3}\sin^3 x - 5 \sin^5 x + C$ | (D) $\frac{1}{3}\sin^3 x + \frac{1}{5} \sin^5 x + C$ |
| (E) $3\sin^3 x - 5 \sin^5 x + C$ | |

السؤال رقم 12 هو تكرار للسؤال رقم 11 و يجب أن تجيب عليه للحصول على درجته

| Q12. $\int \sin^2 x \ \cos^3 x \, dx =$ | |
|---|---|
| (A) $3\sin^3 x - \frac{1}{5} \sin^5 x + C$ | (B) $\frac{1}{3}\sin^3 x - \frac{1}{5} \sin^5 x + C$ |
| (C) $\frac{1}{3}\sin^3 x - 5 \sin^5 x + C$ | (D) $\frac{1}{3}\sin^3 x + \frac{1}{5} \sin^5 x + C$ |
| (E) $3\sin^3 x - 5 \sin^5 x + C$ | |

| Q13. $\int \tan^5 x \ \sec^4 x \ dx =$ | | | |
|---|---|---|---|
| (A) $\frac{\sec^6 x}{6} + \frac{\sec^8 x}{8} + C$ | (B) $\frac{\tan^6 x}{6} + \frac{\tan^5 x}{5} + C$ | (C) $\frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + C$ | (D) $\frac{\tan^6 x}{3} + \frac{\tan^8 x}{4} + C$ |

| Q14. $\int \sin 3x \ \cos x \ dx =$ | |
|---|---|
| (A) $-\frac{\cos 4x}{8} - \frac{\cos 2x}{4} + C$ | (B) $-\frac{\sin 4x}{8} - \frac{\cos 2x}{4} + C$ |
| (C) $-\frac{\cos 4x}{8} - \frac{\sin 2x}{4} + C$ | (D) $-\frac{\sin 4x}{8} - \frac{\sin 2x}{4} + C$ |

Q15.

To evaluate $\int \frac{\sqrt{4-x^2}}{x^2}\,dx$ using Trigonometric substitution, we let

| | |
|---|---|
| (A) $x = 2\tan\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$ | (B) $x = 2\sin\theta,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ |
| (C) $x = 2\sec\theta,\ 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$ | (D) $x = 2\cosh\theta$ |

Q16.

$$\int \frac{\sqrt{4-x^2}}{x^2}\,dx =$$

| | |
|---|---|
| (A) $\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$ | (B) $\frac{\sqrt{4-x^2}}{x} - \tan^{-1}\left(\frac{x}{2}\right) + C$ |
| (C) $-\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$ | (D) $-\frac{\sqrt{4-x^2}}{x} - \tan^{-1}\left(\frac{x}{2}\right) + C$ |

Q17.

To evaluate $\int \frac{\sqrt{x^2-16}}{x^4}\,dx$ using Trigonometric substitution, we let

| | |
|---|---|
| (A) $x = 4\sin\theta,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ | (B) $x = 4\tan\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$ |
| (C) $x = 4\cosh\theta$ | (D) $x = 4\sec\theta,\ 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$ |

Q18.

$$\int \frac{\sqrt{x^2-16}}{x^4}\,dx =$$

| | | | |
|---|---|---|---|
| (A) $\frac{(x^2-16)^{\frac{1}{2}}}{48x^3} + C$ | (B) $\frac{(x^2-16)^{\frac{1}{2}}}{24x^3} + C$ | (C) $\frac{(x^2-16)^{\frac{3}{2}}}{48x^3} + C$ | (D) $\frac{(x^2-16)^{\frac{3}{2}}}{24x^3} + C$ |

Q19.

The form of the partial fraction decomposition of the rational function $\frac{x^2+1}{(x-1)(x^2+2x+2)^2}$ is

| | |
|---|---|
| (A) $\frac{Ax+B}{x-1} + \frac{Cx+D}{x^2+2x+2} + \frac{Ex+F}{(x^2+2x+2)^2}$ | (B) $\frac{A}{x-1} + \frac{B}{x^2+2x+2} + \frac{Cx+D}{(x^2+2x+2)^2}$ |
| (C) $\frac{A}{x-1} + \frac{Bx+C}{x^2+2x+2} + \frac{Dx+E}{(x^2+2x+2)^2}$ | (D) $\frac{A}{x-1} + \frac{Bx+C}{(x^2+2x+2)^2}$ |

Q20.

By using long division, we have $\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} =$

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| $x+1+\frac{4x}{x^3-x^2-x+1}$ | $x-1+\frac{4x}{x^3-x^2-x+1}$ | $x+\frac{4x}{x^3-x^2-x+1}$ | $1-x+\frac{4x}{x^3-x^2-x+1}$ |

Q21.

The form of the partial fraction decomposition of the remainder is $\frac{4x}{x^3-x^2-x+1} = \frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$. Solving for $A, B,$ and $C,$ we get

| (A) | (B) | (C) | (D) |
|---|---|---|---|
| $A=-1, B=-2, C=-1$ | $A=1, B=-2, C=-1$ | $A=-1, B=2, C=1$ | $A=1, B=2, C=-1$ |

Q22.

$$\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx =$$

| (A) | (B) |
|---|---|
| $\frac{x^2}{2}-x+\ln\lvert x-1\rvert+\frac{2}{x-1}-\ln\lvert x+1\rvert+C$ | $\frac{x^2}{2}+x+\ln\lvert x-1\rvert-\frac{2}{x-1}-\ln\lvert x+1\rvert+C$ |
| (C) | (D) |
| $\frac{x^2}{2}-\ln\lvert x-1\rvert+\frac{2}{x-1}+\ln\lvert x+1\rvert+C$ | $x-\frac{x^2}{2}+\ln\lvert x-1\rvert+\frac{2}{x-1}-\ln\lvert x+1\rvert+C$ |

Q23.

$$\int \frac{3^x \cdot \ln 3}{1+3^{2x}}\,dx =$$

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\tan^{-1}(3^x)+C$ | $\sin^{-1}(3^x)+C$ | $\frac{3^{x+1}}{(x+1)\ln 3}+C$ | $\frac{3^{(x+1)}\ln 3}{x+1}+C$ | $\frac{(2x)3^{(x+1)}}{(x+1)\ln 3}+C$ |

السؤال رقم 24 هو تكرار للسؤال رقم 23 و يجب أن تجيب عليه للحصول على درجته

Q24.

$$\int \frac{3^x \cdot \ln 3}{1+3^{2x}}\,dx =$$

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\tan^{-1}(3^x)+C$ | $\sin^{-1}(3^x)+C$ | $\frac{3^{x+1}}{(x+1)\ln 3}+C$ | $\frac{3^{(x+1)}\ln 3}{x+1}+C$ | $\frac{(2x)3^{(x+1)}}{(x+1)\ln 3}+C$ |

Q25.

$$\int \frac{dx}{\sqrt{10x-x^2}} =$$

Hint: complete the square.

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{1}{\sqrt{10x}}+\sin^{-1}x+C$ | $\tan^{-1}(x-5)+C$ | $\sin^{-1}(x-5)+C$ | $\tan^{-1}\left(\frac{x-5}{5}\right)+C$ | $\sin^{-1}\left(\frac{x-5}{5}\right)+C$ |

السؤال رقم 26 هو تكرار للسؤال رقم 25 و يجب أن تجيب عليه للحصول على درجته

Q26.

$$\int \frac{dx}{\sqrt{10x - x^2}} =$$

Hint: complete the square.

| (A) | (B) | (C) | (D) | (E) |
|---|---|---|---|---|
| $\frac{1}{\sqrt{10x}} + \sin^{-1} x + C$ | $\tan^{-1}(x-5) + C$ | $\sin^{-1}(x-5) + C$ | $\tan^{-1}\left(\frac{x-5}{5}\right) + C$ | $\sin^{-1}\left(\frac{x-5}{5}\right) + C$ |

Q27.

$$\int \tan x \ dx =$$

| (A) | (B) | (C) |
|---|---|---|
| $\sec^2 x + C$ | $\frac{\tan^2 x}{2} + C$ | $\ln\lvert\sec x\rvert + C$ |
| (D) | (E) | |
| $\ln\lvert\sin x\rvert + C$ | $\ln\lvert\sec x + \tan x\rvert + C$ | |

السؤال رقم 28 هو تكرار للسؤال رقم 27 و يجب أن تجيب عليه للحصول على درجته

Q28.

$$\int \tan x \ dx =$$

| (A) | (B) | (C) |
|---|---|---|
| $\sec^2 x + C$ | $\frac{\tan^2 x}{2} + C$ | $\ln\lvert\sec x\rvert + C$ |
| (D) | (E) | |
| $\ln\lvert\sin x\rvert + C$ | $\ln\lvert\sec x + \tan x\rvert + C$ | |

Q29.

$$\int \sin^2 x \ \cos x \ \ln(\sin x) \ dx =$$

Hint: let $u = \sin x$ and use a suitable formula

| (A) | (B) |
|---|---|
| $\frac{4}{7} e^{2x} \sin 3x - \frac{2}{7} e^{2x} \cos 3x + C$ | $\frac{1}{9} \sin^3 x \ [3 \ln(\sin x) - 1] + C$ |
| (C) | (D) |
| $\frac{2}{7} e^{2x} \sin 3x - \frac{4}{7} e^{2x} \cos 3x + C$ | $\frac{1}{6} \sin^3 x \ [6 \ln(\sin x) - 1] + C$ |
| (E) | |
| $\frac{\sin^3 x}{3} \cdot \frac{\cos^2 x}{2} + C$ | |

السؤال رقم 30 هو تكرار للسؤال رقم 29 و يجب أن تجيب عليه للحصول على درجته

| | |
|---|---|
| Q30. $\int \sin^2 x \cos x \ln(\sin x)\, dx =$ Hint: let $u = \sin x$ and use a suitable formula | |
| (A) $\frac{4}{7}e^{2x}\sin 3x - \frac{2}{7}e^{2x}\cos 3x + C$ | (B) $\frac{1}{9}\sin^3 x\,[3\ln(\sin x) - 1] + C$ |
| (C) $\frac{2}{7}e^{2x}\sin 3x - \frac{4}{7}e^{2x}\cos 3x + C$ | (D) $\frac{1}{6}\sin^3 x\,[6\ln(\sin x) - 1] + C$ |
| (E) $\frac{\sin^3 x}{3} \cdot \frac{\cos^2 x}{2} + C$ | |

Answers to the second exam

16 / 1 / 1433H

# A

| | |
|---|---|
| 1 | C |
| 2 | C |
| 3 | E |
| 4 | E |
| 5 | D |
| 6 | D |
| 7 | A |
| 8 | A |
| 9 | B |
| 10 | D |
| 11 | B |
| 12 | B |
| 13 | C |
| 14 | A |
| 15 | B |
| 16 | C |
| 17 | D |
| 18 | C |
| 19 | C |
| 20 | A |
| 21 | D |
| 22 | B |
| 23 | A |
| 24 | A |
| 25 | E |
| 26 | E |
| 27 | C |
| 28 | C |
| 29 | B |
| 30 | B |