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Symbol Index

To shorten and unify notation, in this index we use the following convention:

- \( B \) denotes a bipartite (di)graph.
- \( C, C_i \) denote cycles (directed, undirected, edge-coloured, oriented).
- \( D, D_i \) denote digraphs, directed multigraphs and directed pseudographs.
- \( G, G_i \) denote undirected graphs and undirected multigraphs.
- \( H \) denotes a hypergraph.
- \( M \) denotes a mixed graph or a matroid.
- \( P, P_i \) denote path (directed, undirected, edge-coloured, oriented).
- \( S \) denotes a matrix or a multiset.
- \( X, X_i \) denote abstract sets or sets of vertices.
- \( Y, Y_i \) denote sets of arcs.

\[(D_1, D_2)_D: \text{set of arcs with tails in } V(D_1) \text{ and heads in } V(D_2), \] 6
\[(X,\prec): \text{partial order on } X, 236\]
\[(X_1, X_2)_D: \text{set of arcs with tail in } X_1 \text{ and head in } X_2, 3\]
\[(\Gamma, +): \text{an additive group}, 435\]
\[(\bar{K}_n, c): \text{weighted complete digraph}, 82\]
\[(\mathcal{F}, b): \text{pair of a family } \mathcal{F} \text{ and a submodular function } b \text{ on } \mathcal{F}, 449\]
\[*P: \text{ } P \text{ minus the first vertex on } P, 322\]
\[>_{u}: \text{ordering of neighbours of } u, 654\]
\[A(D): \text{arc set of } D, 2\]
\[A(x): \text{arc set of residual network w.r.t } x, 98\]
\[B = (X_1, X_2; E): \text{specification of a bipartite graph with bipartition } X_1, X_2, 25\]
\[BG(D): \text{bipartite representation of } D, 25\]
\[BOL(D): \text{proper backward rank of } D, 643\]
\[BE: \text{bad vertices with respect to the local median order } L, 639\]
\[CM(D): \text{the 2-edge-coloured bipartite multigraph corresponding to the bipartite digraph } D, 602\]
\[CM^{-1}(B): \text{the bipartite digraph corresponding to the 2-edge-coloured bipartite multigraph } B, 602\]
\[C[x_i, x_j]: \text{subpath of } C \text{ from } x_i \text{ to } x_j, 12\]
\[C_1 \gg C_2: \text{ } C_1 \text{ contains singular vertices with respect to } C_2 \text{ and they all are out-singular, and } C_2 \text{ has singular vertices with respect to } C_1 \text{ and they all are in-singular}, 254\]
\[D(G): \text{digraph obtained from } G \text{ via BD-correspondence}, 602\]
\[D(d, n, q, r): \text{consecutive-}d \text{ digraph}, 190\]
\[D - X: \text{deleting the vertices of } X \text{ from } D, 7\]
\[D - Y: \text{deleting the arcs of } Y \text{ from } D, 7\]
\[D//P: \text{path-contraction}, 229\]
\[D/D_1: \text{contracting the subdigraph } D_1 \text{ in } D, 7\]
\[D = (V + s, A), D = (V + s, E \cup F): \text{specification of } D \text{ with special vertex } s, 356\]
\[D = (V, A): \text{specification of } D, 2\]
\[D = (V, A, c): \text{specification of a paired comparison digraph}, 643\]
\[D = (V, A, c): \text{specification of weighted } D, 643\]
\[D_1, D_2, \ldots, D_n: \text{composing } D \text{ with } D_1, D_2, \ldots, D_n, 8\]
\[D^p: \text{pth power of } D, 9\]
$D_1 \Rightarrow D_2$: no arc from $V(D_2)$ to $V(D_1)$, 6

$D_1 \cong D_2$, $D \Rightarrow H$: $D_1$ is isomorphic to $D_2$, 7

$D_1 \ ou \ D_2$: union of $D_1$ and $D_2$, 10

$D_1 \rightarrow D_2$: $D_1$ is homomorphic to $D_2$, 656

$D_1 \not\rightarrow D_2$: $D_1$ is not homomorphic to $D_2$, 656

$D_1 \rightarrow D_2$: $V(D_1)$ dominates $V(D_2)$, 6

$D_1 \times D_2 \times \ldots \times D_n$, $\prod_{i=1}^{n} D_i$: Cartesian product of digraphs, 9

$D_{d,t}(d,t)$: de Bruijn digraph, 187

$D_{d,t}(d,t)$: generalized de Bruijn digraph, 190

$D_{K}(d,t)$: the Kautz digraph, 189

$D_{ST}(d,t)$: digraph obtained from $D$ by the vertex splitting procedure, 102

$D_{x}(d,t)$: digraph associated with a 2-SAT expression, 36

$D_{major}(d,t)$: majority digraph of the hypertournament $T$, 628

$D(X)$: subdigraph of $D$ induced by $X$, 5

$E(G)$: edge set of the graph $G$, 18

$FOR(D)$: proper forward rank of $D$, 643

$F_{s}^{+}$, $F_{s}^{-}$: out- and in-branching rooted at $s$, 19

$G = (V, E)$: specification of the directed graph with special vertex $s$, 440

$GF(2)$: Galois field on 2 elements, 544

$G_{1} \times G_{2} \times \ldots \times G_{n}$: Cartesian product of graphs, 71

$G_{e}$: good vertices with respect to the local median order $L$, 639

$G_{id}$: graph corresponding to orientability as a locally in-tournament digraph, 423

$G_{id}$: graph corresponding to orientability as a locally tournament digraph, 418

$G_{qtd}$: graph corresponding to orientability as a quasi-transitive digraph, 414

$H = (V, E)$: specification of the hypergraph $H$, 24

$K_{n}$: $n$-edge-coloured complete graph of order $n$, 611

$K_{n}$: complete graph of order $n$, 25

$K_{n_{1}, n_{2}, \ldots, n_{r}}$: complete multipartite graph, 25

$L(D)$: line digraph of $D$, 182

$L^{k}(D)$: iterated line digraph of $D$, 187

$M = (S, T)$: specification of matroid, 663

$M = (V, A, E)$: specification of the mixed graph $M$, 22

$MOR(D)$: proper mutual rank of $D$, 643

$M^{*}$: dual of the matroid $M$, 665

$M_{1} \vee M_{2}$: union of matroids $M_{1}$ and $M_{2}$, 668

$N(T, X)$: assignment neighbourhood of $T$ w.r.t $X$, 85

$N_{p}^{+}(X)$, $N_{p}^{-}(X)$: $p$th out- and in-neighbourhood of $X$, 46

$N_{p}^{+}[X]$, $N_{p}^{-}[X]$: closed $p$th out- and in-neighbourhood of $X$, 46

$N_{p}(v)$: neighbour of $v$, 4

$N_{p}[X]$, $N_{p}(X)$: out-neighbourhood, in-neighbourhood of $X$, 4

$N_{p}^{+}(v)$, $N_{p}^{-}(v)$: out-neighbourhood and in-neighbourhood of $v$, 4

$N_{C}(x)$: neighbour of $x$ in $G$, 19

$O(f(k))$: $O$-notation, 29

$OR(D)$: set of all FSO-optimal orderings of $V$, 642

$P[x_{i}, x_{j}]$: subpath of $P$ from $x_{i}$ to $x_{j}$, $i \leq j$, 12

$Q_{z,x}$, $Q_{w}$: path factor with two paths such that the first is an $(x,z)$-path and the second path has terminal vertex $w$, 295

$Q_{z,x}$, $Q_{w}$: path factor with two paths such that the first is a $(z,x)$-path and the second path has initial vertex $w$, 295

$R^{+}(X)$: vertices that can be reached from $X$, 322

$R^{-}(X)$: vertices that can reach $X$, 322

$R_{1}(r, q)$: Ramsey number for $l$-uniform hypergraphs, 561
Symbol Index 719

$S = [s_{ij}]$: matrix, 2

$SC(D)$: strong component digraph of $D$, 17

$S^T$: transpose of matrix $S$, 2

$TC(D)$: transitive closure of $D$, 177

$TT$: transitive tournament on $s$ vertices, 414

$T^{rev}$: reverse of $T$, 591

$UG(D)$: underlying graph of $D$, 19

$U_{n,k}$: uniform matroid, 664

$V(D)$: vertex set of $D$, 2

$V(G)$: vertex set of the graph $G$, 18

$X^+$, $X^-$: successors and predecessors of vertices in $X$, 12

$X_1 \Rightarrow X_2$: no arc from $X_2$ to $X_1$, 3

$X_1 \rightarrow X_2$: $X_1 \rightarrow X_2$ and $X_1 \Rightarrow X_2$, 3

$X_1 \rightarrow X_2$: $X_1$ dominates $X_2$, 3

$X_1 \times X_2 \times \ldots \times X_p$: Cartesian product of sets, 2

$X_1 \Delta X_2$: symmetric difference, 544

$\Delta(G)$: maximum degree of $G$, 19

$\Delta^+(D)$, $\Delta^-(D)$: maximum out- and in-degree of $D$, 5

$\Delta^0(D)$: maximum semi-degree of $D$, 5

$\Delta_{mon}(G)$: maximum monochromatic degree of $G$, 591

$\Gamma(F)$: intersection graph of the family $F$ of sets, 424

$\Omega(F)$: catch digraph of the family $F$ of pointed sets, 424

$\Omega(f(k))$: \Theta-notation, 29

$\Omega(f(P))$: intersection graph of the family $P$ of subgraphs, 600

$\Omega(D)$: maximum number of arc-disjoint dicuts in $D$, 398

$\Phi^{ext}$: set of extended \Phi-digraphs, 9

$\Phi_0$: union of semicomplete multipartite, connected extended locally semicomplete digraphs and acyclic digraphs, 214

$\Phi_1$: union of semicomplete bipartite, connected extended locally semicomplete and acyclic digraphs, 214

$\Phi_2$: union of connected extended locally semicomplete and acyclic digraphs, 214

$\Psi$: union of transitive and extended semicomplete digraphs, 195

$\Psi_f$: class of all digraphs for which a minimum path-factor can be found in polynomial time $O(n^s)$, 335

$\Theta(f(k))$: \Theta-notation, 29

$\alpha(D)$: independence number of $D$, 22

$\alpha_{acyc}(D)$: acyclic independence number of $D$, 662

$\alpha_m(D)$: oriented independence number of $D$, 662

$\mathcal{I}, \mathcal{J}$: admissible cells for transportation, 149

$\chi(X_1, X_2)$: colour of edges between $X_1$ and $X_2$, 591

$\chi(e)$: colour of edge $e$, 591

$\chi_{end}(P)$: colour of last edge of $P$, 591

$\chi_{start}(P)$: colour of first edge of $P$, 591

$\chi(H)$: chromatic number of $D$, 22

$\delta(G)$: minimum degree of $G$, 19

$\delta^+(D)$, $\delta^-(D)$: minimum out- and in-degree of $D$, 4

$\delta^0(D)$: minimum semi-degree of $D$, 5

$\delta^0_{ij}$: length of a shortest $(i,j)$-path using only internal vertices from $\{1, 2, \ldots, m-1\}$, 58

$\delta_{mon}^0(D)$: minimum monochromatic semi-degree of the arc-coloured digraph $D$, 618

$\delta_{mon}^0(v)$: minimum monochromatic semi-degree of $v$ in an arc-coloured digraph, 618

$\delta(P)$: capacity of augmenting path $P$, 109

$\delta_s(s, t)$: length of a shortest $(s, t)$-path in $\mathcal{N}(x)$, 114

$\epsilon(xy)$: weight of the arc $xy$ in a paired comparison digraph, 641

$\eta_h(F)$: deficiency of the family $F$ of one-way pairs, 367

$\eta_h(X, Y)$: deficiency of the one-way pair $(X, Y)$, 366

$\gamma_{k,s,T}(D)$: $k$-$(S,T)$-arc-strong connectivity augmentation number of $D$, 374

$\gamma(S, \mathcal{F})$: flow demand of the $(s,t)$-cut $(S, \mathcal{F})$, 127

$\gamma^*_{k}(D)$: subpartition lower bound for augmenting the vertex-strong connectivity of $D$ to $k$, 365
\(\gamma_k(D)\): subpartition lower bound for augmenting the arc-strong connectivity of \(D\) to \(k\), 360

\(\gamma_s,k(D)\): minimum number of new arcs one has to add to \(D\) in order to obtain a new digraph \(D' = (V, A \cup F)\) which has \(k\) arc-disjoint out-branchings rooted at \(s\), 534

\(\kappa(D)\): vertex-strong connectivity of \(D\), 16

\(\kappa(x, y)\): local vertex-strong connectivity from \(x\) to \(y\), 344

\(\lambda(D)\): arc-strong connectivity of \(D\), 17

\(\lambda(x, y)\): local arc-strong connectivity from \(x\) to \(y\), 344

\((Y_1, Y_2)\): scalar product of \(Y_1\) and \(Y_2\), 544

\(\dim S\): dimension of the vector space \(S\), 544

\(G\): complete biorientation of \(G\), 19

\(K_n\): complete digraph of order \(n\), 27

\(\mu_D(x, y)\): number of arcs with tail \(x\) and head \(y\), 4

\(\mu_G(u, v)\): number of edges between \(u\) and \(v\) in \(G\), 18

\(\nu_0(D)\): maximum number of vertex-disjoint cycles in \(D\), 551

\(\nu_1(D)\): maximum number of vertex-disjoint cycles in \(D\), 551

\(\overline{G}\): complement of \(G\), 18

\(\overline{K}_n\): graph of order \(n\) with no edges, 25

\(\varphi\): negation of boolean variable \(x\), 35

\(\varphi(u)\): forefather of \(u\), 180

\(\pi_{FSO}(x)\): proper FSO rank of \(x\), 642

\(\rho(G)\): \(\operatorname{dim}_{\min}(G) - \operatorname{diam}(G)\), 67

\(\rho(D)\): minimum number of arcs whose contraction in \(D\) leads to a strong directed multigraph, 399

\(\sigma^+(x)\), \(\sigma^-(x)\): positive and negative scores of \(x\), 641

\(\tau_0(D)\): size of a minimum feedback vertex set of \(D\), 551

\(\tau_1(D)\): size of a minimum feedback arc set of \(D\), 551

\(\tau(D)\): size of a minimum dijoin of \(D\), 398

\(\overline{C}_n\): directed cycle on \(n\) vertices, 12

\(\bar{P}_n\): directed path on \(n\) vertices, 12

\(a_k(D)\): \(k\)-strong augmentation number of \(D\), 366

\(a_F\): the number of edges, oriented or not, which enter some \(X \in \mathcal{F}\), 505

\(b(v)\): balance prescription for the vertex \(v\), 96

\(b_D(a)\): backward length of the ordering \(alpha\), 643

\(b_i\): balance vector of the flow \(x\), 96

\(bd(F)\): boundary of face \(F\), 219

\(c(G)\): the number of connected components of \(G\), 445

\(c(Y)\): sum of costs/weights of arcs in \(Y\), 6

\(c(a)\): cost/weight of the arc \(a\), 6

\(d(X, Y)\): \(d^+(X, Y) + d^-(Y, X)\), 344

\(d(x)\): degree of \(x\), 19

\(d^+(X, Y)\): number of arcs with tail in \(X - Y\) and head in \(Y - X\), 344

\(d^+_F(X)\), \(d^-_F(X)\): number of arcs from \(F\) that leave, respectively enter, \(X\), 474

\(d_D(X)\): degree of \(X\), 4

\(d^+_D(X)\), \(d^-_D(X)\): out- and in-degree of \(X\), 4

\(d^+_F(v),d^-_F(v)\): \(v\)th out- and in-degree of \(v\) in an arc-coloured digraph, 618

\(d_j(v)\): \(j\)th degree of \(v\), 591

\(e(X_1, X_2)\): number of edges between \(X_1\) and \(X_2\), 502

\(e_G(X)\): number of edges of \(G\) with at least one end in \(X\), 445

\(e_F\): number of edges connecting different sets of partition \(\mathcal{F}\), 448

\(f(X_1, X_2)\): sum of \(f\)-values over arcs with tail in \(X_1\) and head in \(X_2\), 96

\(f_D(alpha)\): forward length of the ordering \(alpha\), 643

\(g(D)\): girth of \(D\), 11

\(g_v(D)\): length of a shortest cycle through \(v\) in \(D\), 303

\(h(X, Y)\): number of vertices not in the one-way pair \((X, Y)\), 366

\(h(p)\): height of vertex \(p\), 118

\(\tau_{LC}(X)\): number of edges of \(G\) with both ends in \(X\), 445
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{pq}(D)$</td>
<td>global irregularity of $D$, 262</td>
</tr>
<tr>
<td>$i_l(D)$</td>
<td>local irregularity of $D$, 262</td>
</tr>
<tr>
<td>$l(S, S)$</td>
<td>lower bound of the cut $(S, S)$, 126</td>
</tr>
<tr>
<td>$l_{ij}$</td>
<td>lower bound of the arc $ij$, 95</td>
</tr>
<tr>
<td>$m(y, e)$</td>
<td>sum of values of $y$ on sets sets entered by the arc $e$, 528</td>
</tr>
<tr>
<td>$m_x(x)$</td>
<td>mutual length of ordering $x$, 643</td>
</tr>
<tr>
<td>$p(D)$</td>
<td>period of $D$, 564</td>
</tr>
<tr>
<td>$r(X)$</td>
<td>rank of $X$, 664</td>
</tr>
<tr>
<td>$r^+(U)$</td>
<td>sum of function values of $r$ on arcs in $(U, X)$, 449</td>
</tr>
<tr>
<td>$r^-(U)$</td>
<td>sum of function values of $r$ on arcs in $(X, U)$, 449</td>
</tr>
<tr>
<td>$r_b(D)$</td>
<td>minimum number of arcs to reverse in $D$ to obtain a $k$-strong digraph, 376</td>
</tr>
<tr>
<td>$r_{ij}$</td>
<td>residual capacity of the arc $ij$, 98</td>
</tr>
<tr>
<td>$s(G)$</td>
<td>minimum number of steps for gossiping in $G$, 81</td>
</tr>
<tr>
<td>$sgn(P)$</td>
<td>$-1$ if $P$ is an in-path and $+1$ if $P$ is an out-path, 322</td>
</tr>
<tr>
<td>$u(S, S)$</td>
<td>capacity of the $(s, t)$-cut $(S, S)$, 108</td>
</tr>
<tr>
<td>$u_{ij}$</td>
<td>capacity of the arc $ij$, 95</td>
</tr>
<tr>
<td>$x(S, S)$</td>
<td>flow across the $(s, t)$-cut $(S, S)$, 109</td>
</tr>
<tr>
<td>$x(uv)$</td>
<td>value of integer flow $x$ on the arc $uv$, 435</td>
</tr>
<tr>
<td>$x + x'$</td>
<td>arc-sum of flows $x$ and $x'$, 104</td>
</tr>
<tr>
<td>$x</td>
<td></td>
</tr>
</tbody>
</table><p>ightarrow y$ | $x$ dominates $y$, 3 |
| $x &gt; y$ | $x$ is a descendant of $y$ in a DFS tree, 173 |
| $x^<em>$ | adding the residual flow $\bar{x}$ to $x$, 105 |
| $x^{+}, x^{-}$ | successor and predecessor of $x$, 12 |
| $x_i^{+}, x_i^{-}$ | on the arc $ij$, 96 |
| $A(D)$ | arc space of $D$, 544 |
| $C(D)$ | cycle space of $D$, 544 |
| $C^</em>(D)$ | cocycle space of $D$, 545 |
| $D_0, D_6$ | classes of non-arc-pancyclic arc-3-cyclic tournaments, 309 |
| $F = P_1 \cup \ldots \cup P_q \cup C_1 \cup \ldots \cup C_l$ | $q$-path-cycle subdigraph, 15 |
| $N(D)$ | network representation of $D$, 346 |
| $N(x)$ | residual network w.r.t $x$, 98 |</p>

Symbol Index 721

$N = (V, A, l, u, b, c)$: specification of the flow network $N$, 96

$N_B$: network corresponding to the bipartite graph $B$, 138

$N_S = (V, A, f, g, (B, b), c)$: submodular flow network, 454

$N_{(\alpha, \beta)}$: admissible network with respect to $(\alpha, \beta)$, 151

$Q$: set of rational numbers, 1

$Q_+$: set of positive rational numbers, 1

$Q_0$: set of non-negative rational numbers, 1

$R$: set of reals, 1

$R_+$: set of positive reals, 1

$R_0$: set of non-negative reals, 1

$S \leq_T T$: $S$ polynomially reducible to $T$, 34

$T^*$: set of second powers of even cycles of length at least 4, 290

$T_1, T_6$: classes of semicomplete digraphs, 290

$Z$: set of integers, 1

$Z_+$: set of positive integers, 1

$Z_0$: set of non-negative integers, 1

$\text{Prob}(E)$: probability of the event $E$, 548

$\text{diam}(D)$: diameter of $D$, 47

$\text{diam}_\text{min}(G)$: minimum diameter of an orientation of $G$, 63

$\text{dist}(X_1, X_2)$: distance from $X_1$ to $X_2$, 47

$\text{dist}(x, y)$: distance from $x$ to $y$, 47

$\text{domn}(A, n)$: domination number of heuristic $A$ for TSP problem of order $n$, 336

$\text{ext}(X)$: set of elements each of which can extend $X$ to an independent set, 666

$\text{in}(D)$: intersection number of $D$, 217

$\text{lc}(D)$: length of a longest cycle of $D$, 575

$\text{lp}(D)$: length of a longest path in $D$, 433

$\text{lp}(G)$: longest path in $G$, 61

$\text{pcc}(D)$: path-cycle covering number of $D$, 15

$\text{pcc}^*(D)$: $0$ if $D$ has a cycle factor and $\text{pcc}(D)$ otherwise, 331

$\text{pc}(D)$: path covering number of $D$, 15
pc_x(D): minimum number of paths in a path factor which starts at x, 283
pc^*(D): 0 if D is hamiltonian and pc(D) otherwise, 333
ph(D): pseudo-hamiltonic number of D, 232
pred(x): predecessor of x w.r.t a DFS search, 172
qhn(D): quasi-hamiltonic number of D, 230
rad(D): radius of D, 47
rad^+(D): out-radius of D, 47
rad^−(D): in-radius of D, 47
srad(D): strong radius of D, 64
texpl(x): time when x is explored by a DFS search, 172
tvisit(x): time when x is visited in a DFS search, 172
|D|: the order of the digraph D, 2
|S|: cardinality of the multiset S, 2
|x|: value of flow x, 100
co-NP: class of co-NP decision problems, 33
NP: class of NP decision problems, 33