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Stochastic Analysis of Subsurface Heterogeneity Using Markov Chains: A Case Study in the Netherlands.

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ABSTRACT

This research presents an application of specific types of a Markov chain model (called the coupled Markov chain model) to characterize subsurface heterogenity. This model incorprates soft geological data in additional to the available hard data to charactrize subsurface heterogenity conditioned on a number of boreholes. Data from resarch site in the Netherlands are used to evaluate the performance of the coupled Markov chain model (CMC). The model is used to study the effect of conditioning on number of boreholes on reducing the uncertainty of the subsurface heterogenity at the site.

INTRODUCTION

Groundwater contamination becomes an important environmental issue which possesses a serious threat to drinking water quality. Many studies had focused on minimizing concentration uncertainty by conditioning the hydraulic conductivity fields either on measurements of hydraulic conductivity by solving the forward groundwater flow and transport problems, Van Leeuwen [2000], or conditioning on groundwater head data and concentration data in solving the inverse groundwater flow and transport problems; Zimmerman et al. [1996] and Valstar [2001]. In this paper, the forward approach is followed. However, conditionning is preformed on geological data rather than on measurments of hydraulic conductivity.

This paper presents an application of the coupled Markov chain (CMC) developed by Elfeki and Dekking [2001] on a site in the Netherlands.

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The study area is located in the central Rhine-Meuse delta in the Netherlands where intensive measurements are available on subsurface data. A comparison is made between the coupled Markov chain technique used in this study and the sequential indicator simulation technique used by Bierkens [1994] to simulate the geological section.

THE COUPLED MARKOV CHAIN ON 2D-DOMAIN

Elfeki and Dekking [2001] have developed the coupled Markov chain theory. The two-coupled one-dimensional Markov chains (X_i) that describe the variation in lithologies in the horizontal direction, and (Y_j) that describes the variation in lithologies in the vertical direction is used to construct a two dimensional spatial stochastic process on a lattice ($Z_{i,j}$), Figure (1).

In their description they supposed a two-dimensional domain of cells in which each cell has a row number j and a column number i. They also considered a given number of geological materials or states, n_s (n_s = 6). These materials are coded in numbers, [Elfeki and Dekking, 2001]. They conditioned the coupled Markov chain on multiple wells. They followed an approximation. They conditioned the horizontal chain first, and coupled the conditioned horizontal chain with the vertical one.

The stochastic process ($Z_{i,j}$) could be obtained by coupling the (X_i) and (Y_j) chains. These chains are forced to realize the same states from both directions, the following equation is used in applying conditioned coupled Markov chain on two wells,

$$\Pr\left(\mathrm{Z}_{\mathrm{i,j}} = S_k \middle| \mathrm{Z}_{i-1,j} = S_l, \mathrm{Z}_{i,j-1} = S_m, \mathrm{Z}_{N_{x},j} = S_q\right) =$$
$$\mathrm{C}'\Pr\left(\mathrm{Z}_{\mathrm{i,j}} = S_k \middle| \mathrm{Z}_{i-1,j} = S_l, \mathrm{Z}_{N_{x},j} = S_q\right).\Pr\left(\mathrm{Z}_{i,j} = S_k \middle| \mathrm{Z}_{i,j-1} = S_m\right) \qquad (1)$$

Where C' is the normalizing constant which arises by not permitting transitions in the (X_i) and (Y_j) chains to different states. To apply the coupled Markov chain model, the set of possible states of the system should be identified. Transition probabilities should be computed either from boreholes or from geological maps derived from similar sites. After estimating horizontal and vertical transition matrices, conditional simulation procedure is applied on two neighboring wells, Figure (1). Well data is inserted at their locations starting from the left side of the domain at (1, j), j = 2,…, N_y (well 1) then to the next well (well 2), Figure (2). The procedure of conditioning starts from left to right. The first and second wells are used to generate the domain between them, then the second and third wells are used to generate the domain between them and so on. The procedure stops after all the domains are filled with states.

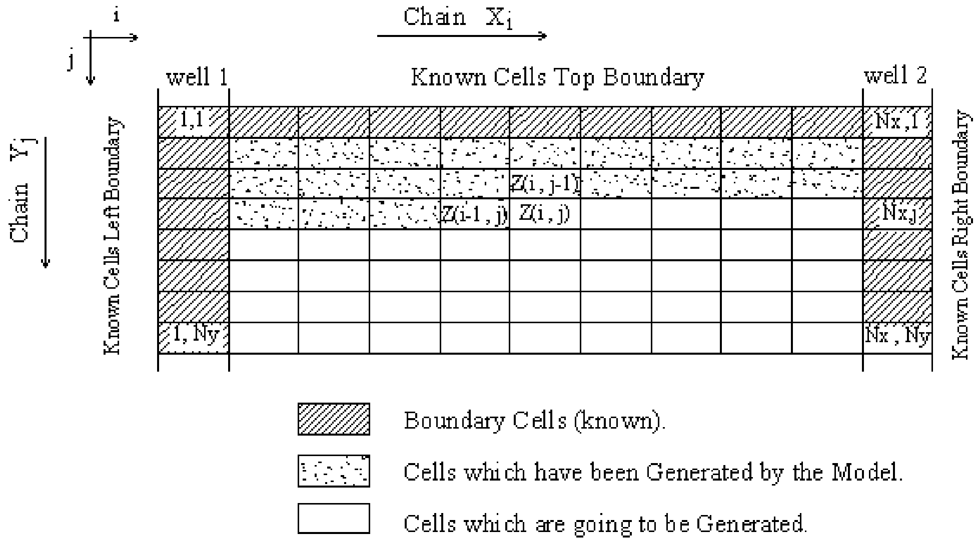

Figure 1. Conditioned coupled Markov chain on the right boundary.

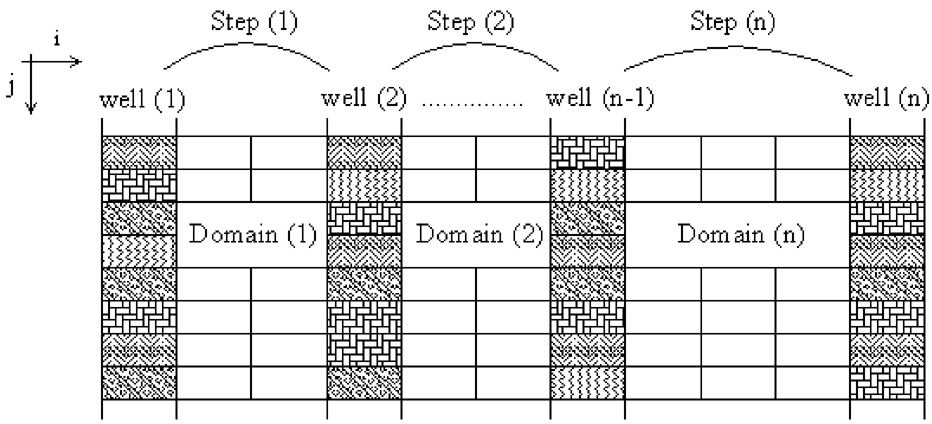

Figure 2. Procedure of generating the domain between successive wells.

The Netherlands Case Study

The study area is located in the central Rhine-Meuse delta in the Netherlands. The section is 1650 m long and 10 m in depth. Data of many cheap hand augured drillings are available, which describe the vertical sequence of the confining layers at boreholes. In addition to these drillings, a few measurements of hydraulic conductivity and porosity are performed on sediment cores. Merging soft geological information (which may be available from geological maps, boreholes data, and geological expertise) with a few hard hydraulic data (such as direct measurements of flow and transport parameters e.g. hydraulic conductivity, porosity, dispersivity,…etc.) make an estimation for the hydraulic properties of the entire confining layer at the scale of interest. six lithogenetic units are distinguished; channel deposits (sand), natural levee deposits (fine sand, sandy clay, silty clay), crevasse splay deposits (fine sand, sandy clay, silty clay), flood basin deposits (clay, humic clay), and peat. Figure 3 shows the study area drawn from 81 boreholes, [Bierkens 1994].

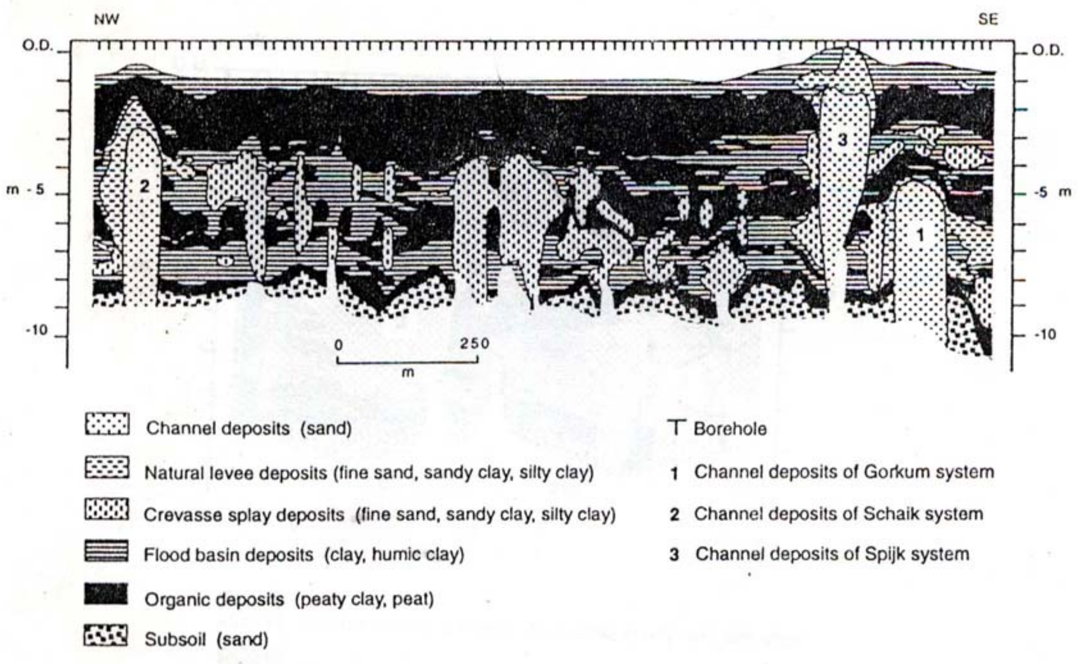

Figure 3. The study area drawn from 81 boreholes, [Bierkens 1994].

The transition probability matrices of the coupled Markov chain model are calculated in both horizontal and vertical directions from the image and displayed in Table 1. These transitions are used to generate realizations of the geological structure. In Table 1 the sampling intervals in both X- and y-directions are given.

Table 1 Data of the geological section and the estimated transition probabilities from the image (1650 x 10 m)

Length of the original section = 1650 m	Depth of the original section = 10 m
Sampling interval in X-direction = 1.5625m	Sampling interval in Y-direction = 0.25m
No. of states = 6	

	Horizontal transition probability matrix						Vertical transition probability matrix					
State	1	2	3	4	5	6	1	2	3	4	5	6
1	.979	.004	.001	.006	.009	.001	.945	.000	.009	.000	.009	.037
2	.020	.965	.001	.008	.006	.000	.071	.796	.021	.041	.071	.000
3	.003	.002	.966	.013	.016	.000	.000	.000	.797	.086	.089	.028
4	.000	.001	.009	.983	.007	.000	.003	.013	.041	.714	.222	.007
5	.001	.001	.006	.007	.984	.001	.004	.012	.047	.119	.768	.050
6	.000	.000	.001	.000	.002	.997	.000	.000	.000	.000	.000	1.00

Two simulations were performed on the original section to evaluate the effect of increasing the number of conditioning boreholes on the improvement of the geological realization. Table 2 shows simulation scenarios.

Table 2 Simulation scenarios.

No.	No. of boreholes used	Spacing (m)
1	20	88
2	39	44

Figure 4 shows a single realization and images of the ensemble average over 50 realizations of the indicator functions of six different lithologies conditioned on twenty boreholes. These boreholes are 88 m apart. The discrepancies between the original section and the simulated one are due to the fact that conditionning is made only on twenty boreholes.

The same Figure shows also the indicator function of each state (The indicator function of each state can be considered as a measure of the probability of occurrence of a specific lithology at specific location. If the indicator function of the state equals one that means, that it is 100% sure that the lithology is located at that position). The indicator function is the gray scale ranges from 0 to 1 where if the indicator function of the state=1 then the state is located at this position and if the indicator function of the state=0 then the state is not located at this position.

It is obvious that the single realization captures many features of the geological image specially top and bottom layers. However, some geological bodies are not completely reproduced such as state 1 appears in black.

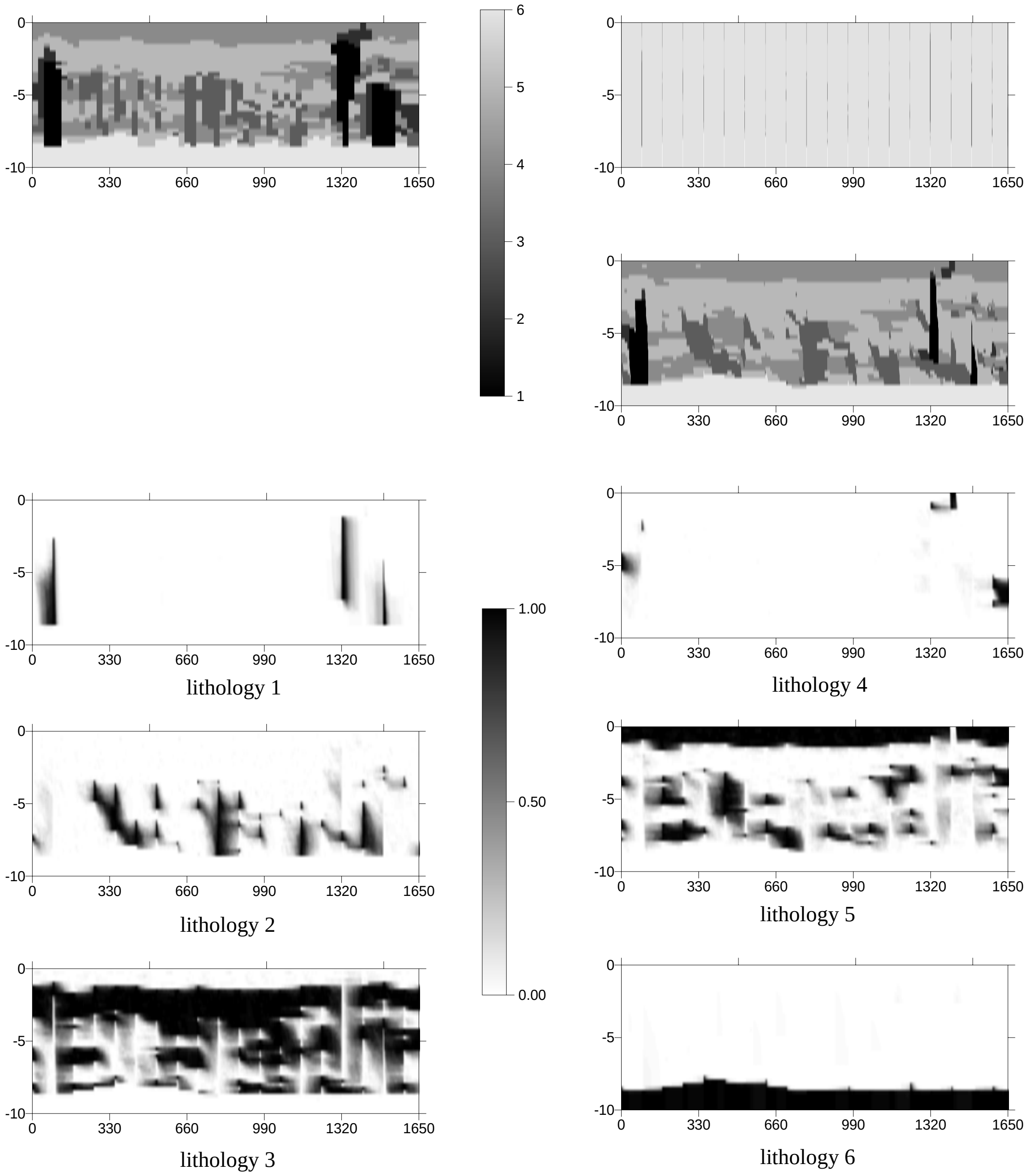

Figure 4. Single realization and ensemble average over 50 realizations conditioned on 20 boreholes. Top left is the real geological structure (original section), top right is boreholes location, the second row (right image) is a single realization generated by the model, the rest of rows are the ensemble average of the indicators of each lithology (probability of occurance).

Another simulation performed on 39 boreholes. Figure 5 shows the image of the ensemble average over 50 realizations. These boreholes are 44 m apart. Almost all the geological features at the site are captured.

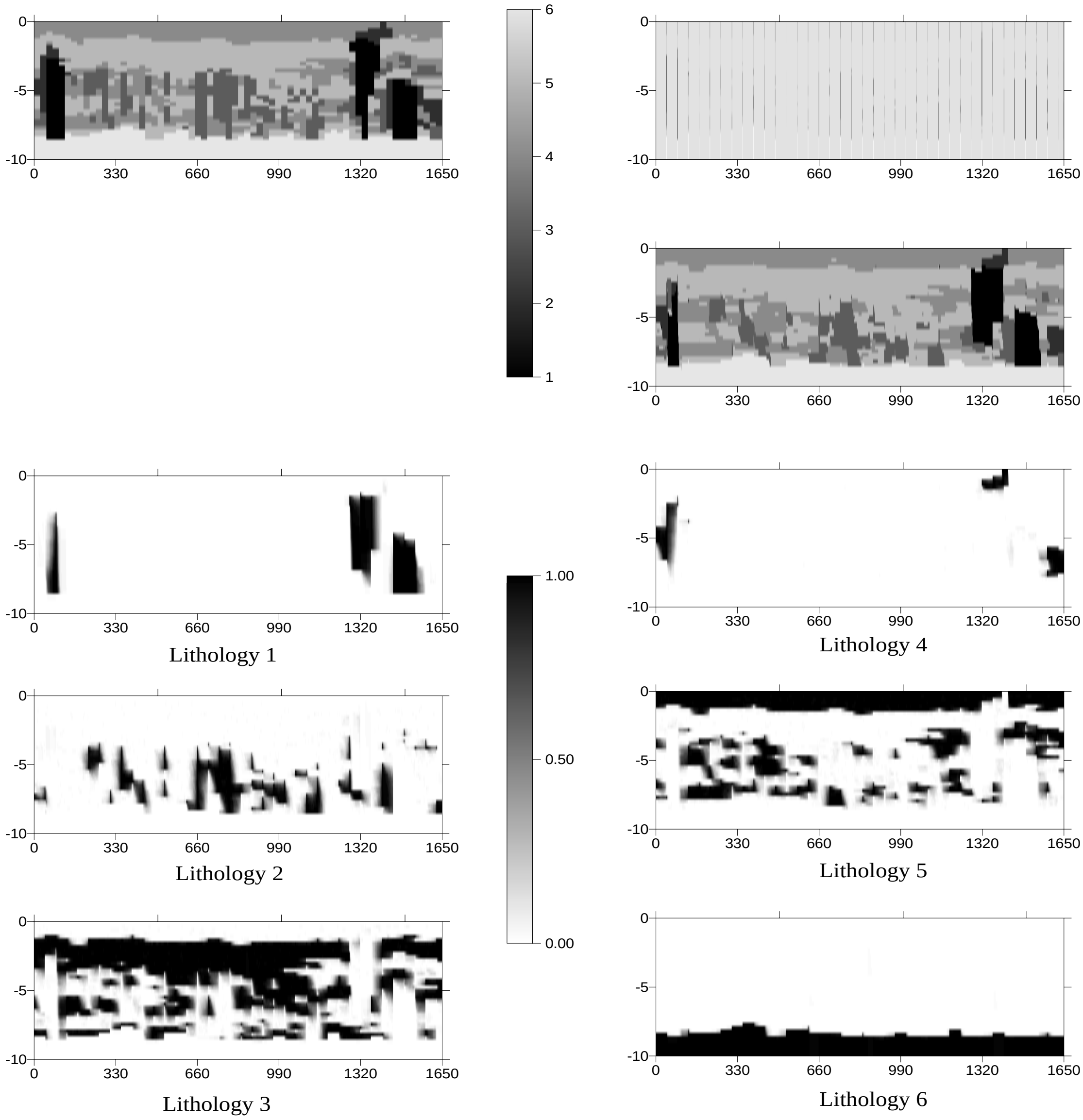

Figure 5. Single realization and ensemble average over 50 realizations conditioned on 20 boreholes. Top left is the real geological structure (original section), top right is boreholes location, the second row (right image) is a single realization generated by the model, the rest of rows are the ensemble average of the indicators of each lithology (probability of occurance).

It is worth mentioning that Bierkens [1994] used 81 boreholes to simulate the same section using sequential indicator technique. However, with the coupled Markov model, it was successful to reach nearly the same results by only 39 boreholes (almost half the number used by Bierkens [1994]), Figure 6 shows the simulation image by Bierkens [1994] using the sequential indicator technique conditioned on 81 boreholes and the one simulated using the coupled Markov chain model.

Figure 6-3 in the thesis p. 83

Figure 6. The top image is the real geological structure (original section), the second image is a single realization generated by the model conditioned on 39 boreholes, the third image shows the simulation by Bierkens [1994] using the sequential indicator technique conditioned on 81 boreholes. (Note: The gray scale in Bierkens is different from the one used in this study)

CONCLUSIONS

The coupled Markov chain model proved to be a valuable tool in predicting the configuration of the heterogeneous geological structures. Conditioning is among the greatest strengths of the coupled Markov chain method. The geological knowledge gained from boreholes can help in producing plausible realizations of the continuity and discontinuity of lithologies.

The coupled Markov chain model (CMC) has shown successful results when applied at the central Rhine-Meuse delta in the Netherlands, using 39 boreholes. The CMC technique conditioned on 39 boreholes gives almost similar results when compared with the sequential indicator simulation (SIS) technique used by Bierkens [1994] conditioned on 81 boreholes (i.e. almost double the number of boreholes used by the CMC technique). It is important to mention that Bierkens handled some parts of the image in a deterministic sense; however, in the (CMC) method the whole image was treated stochastically. This gives more superurity in (CMC) method.

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