CHAPTER

3

PRESSURE AND FLUID STATICS

This chapter deals with forces applied by fluids at rest or in rigid-body motion. The fluid property responsible for those forces is *pressure,* which is a normal force exerted by a fluid per unit area. We start this chapter with a detailed discussion of pressure, including *absolute* and *gage pressures*, the pressure at a *point*, the *variation of pressure with depth* in a gravitational field, the *manometer*, the *barometer*, and pressure measurement devices. This is followed by a discussion of the *hydrostatic forces* applied on submerged bodies with plane or curved surfaces. We then consider the *buoyant force* applied by fluids on submerged or floating bodies, and discuss the *stability* of such bodies. Finally, we apply Newton's second law of motion to a body of fluid in motion that acts as a rigid body and analyze the variation of pressure in fluids that undergo linear acceleration and in rotating containers. This chapter makes extensive use of force balances for bodies in static equilibrium, and it will be helpful if the relevant topics from statics are first reviewed.

OBJECTIVES

When you finish reading this chapter, you should be able to

- Determine the variation of pressure in a fluid at rest
- Calculate the forces exerted by a fluid at rest on plane or curved submerged surfaces
- Analyze the rigid-body motion of fluids in containers during linear acceleration or rotation

3–1 ▪ PRESSURE

Pressure is defined as *a normal force exerted by a fluid per unit area*. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is *normal stress*. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a **pascal** (Pa). That is,

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

The pressure unit pascal is too small for pressures encountered in practice. Therefore, its multiples *kilopascal* ($1 \text{ kPa} = 10^3 \text{ Pa}$) and *megapascal* ($1 \text{ MPa} = 10^6 \text{ Pa}$) are commonly used. Three other pressure units commonly used in practice, especially in Europe, are *bar, standard atmosphere,* and *kilogram-force per square centimeter:*

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$\begin{aligned} 1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} \\ &= 0.9807 \text{ bar} \\ &= 0.9679 \text{ atm} \end{aligned}$$

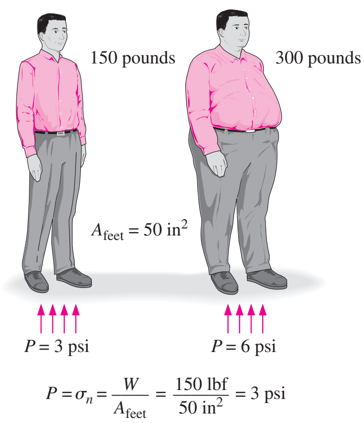


FIGURE 3–1
The normal stress (or "pressure") on the feet of a chubby person is much greater than on the feet of a slim person.

Note the pressure units bar, atm, and kgf/cm^2 are almost equivalent to each other. In the English system, the pressure unit is *pound-force per square inch* (lbf/in^2, or psi), and $1 \text{ atm} = 14.696$ psi. The pressure units kgf/cm^2 and lbf/in^2 are also denoted by kg/cm^2 and lb/in^2, respectively, and they are commonly used in tire gages. It can be shown that $1 \text{ kgf/cm}^2 = 14.223$ psi.

Pressure is also used for solids as synonymous to *normal stress,* which is force acting perpendicular to the surface per unit area. For example, a 150-pound person with a total foot imprint area of 50 in^2 exerts a pressure of $150 \text{ lbf}/50 \text{ in}^2 = 3.0$ psi on the floor (Fig. 3–1). If the person stands on one foot, the pressure doubles. If the person gains excessive weight, he or she is likely to encounter foot discomfort because of the increased pressure on the foot (the size of the foot does not change with weight gain). This also explains how a person can walk on fresh snow without sinking by wearing large snowshoes, and how a person cuts with little effort when using a sharp knife.

FIGURE 3–2
Two basic pressure gages.
Dresser Instruments, Dresser, Inc. Used by permission.

The actual pressure at a given position is called the **absolute pressure,** and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere (Fig. 3–2), and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the **gage pressure.** Pressures below atmospheric pressure are called **vacuum pressures** and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are all positive quantities and are related to each other by

$$P_{gage} = P_{abs} - P_{atm} \tag{3–1}$$

$$P_{vac} = P_{atm} - P_{abs} \tag{3–2}$$

This is illustrated in Fig. 3–3.

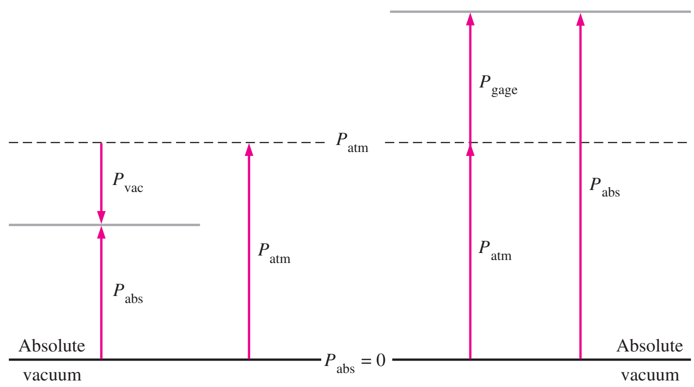


FIGURE 3–3
Absolute, gage, and vacuum pressures.

Like other pressure gages, the gage used to measure the air pressure in an automobile tire reads the gage pressure. Therefore, the common reading of 32 psi (2.25 kgf/cm^2) indicates a pressure of 32 psi above the atmospheric pressure. At a location where the atmospheric pressure is 14.3 psi, for example, the absolute pressure in the tire is 32 + 14.3 = 46.3 psi.

In thermodynamic relations and tables, absolute pressure is almost always used. Throughout this text, the pressure P will denote *absolute pressure* unless specified otherwise. Often the letters "a" (for absolute pressure) and "g" (for gage pressure) are added to pressure units (such as psia and psig) to clarify what is meant.

EXAMPLE 3–1 **Absolute Pressure of a Vacuum Chamber**

A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

SOLUTION The gage pressure of a vacuum chamber is given. The absolute pressure in the chamber is to be determined.
Analysis The absolute pressure is easily determined from Eq. 3–2 to be

$$P_{abs} = P_{atm} - P_{vac} = 14.5 - 5.8 = \mathbf{8.7\ psi}$$

Discussion Note that the *local* value of the atmospheric pressure is used when determining the absolute pressure.

Pressure at a Point

Pressure is the *compressive force* per unit area, and it gives the impression of being a vector. However, pressure at any point in a fluid is the same in all directions. That is, it has magnitude but not a specific direction, and thus it is a scalar quantity. This can be demonstrated by considering a small wedge-shaped fluid element of unit length (into the page) in equilibrium, as shown in Fig. 3–4. The mean pressures at the three surfaces are P_1, P_2, and P_3, and the force acting on a surface is the product of mean pressure and the

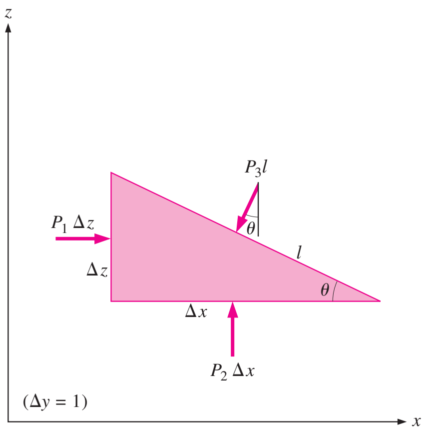


FIGURE 3–4
Forces acting on a wedge-shaped fluid element in equilibrium.

surface area. From Newton's second law, a force balance in the x- and z-directions gives

$$\sum F_x = ma_x = 0: \qquad P_1\,\Delta z - P_3 l \sin\theta = 0 \tag{3–3a}$$

$$\sum F_z = ma_z = 0: \qquad P_2\,\Delta x - P_3 l \cos\theta - \frac{1}{2}\rho g\,\Delta x\,\Delta z = 0 \tag{3–3b}$$

where ρ is the density and $W = mg = \rho g\,\Delta x\,\Delta z/2$ is the weight of the fluid element. Noting that the wedge is a right triangle, we have $\Delta x = l\cos\theta$ and $\Delta z = l\sin\theta$. Substituting these geometric relations and dividing Eq. 3–3a by Δz and Eq. 3–3b by Δx gives

$$P_1 - P_3 = 0 \tag{3–4a}$$

$$P_2 - P_3 - \frac{1}{2}\rho g\,\Delta z = 0 \tag{3–4b}$$

The last term in Eq. 3–4b drops out as $\Delta z \to 0$ and the wedge becomes infinitesimal, and thus the fluid element shrinks to a point. Then combining the results of these two relations gives

$$P_1 = P_2 = P_3 = P \tag{3–5}$$

regardless of the angle θ. We can repeat the analysis for an element in the xz-plane and obtain a similar result. Thus we conclude that *the pressure at a point in a fluid has the same magnitude in all directions*. It can be shown in the absence of shear forces that this result is applicable to fluids in motion as well as fluids at rest.

Variation of Pressure with Depth

It will come as no surprise to you that pressure in a fluid at rest does not change in the horizontal direction. This can be shown easily by considering a thin horizontal layer of fluid and doing a force balance in any horizontal direction. However, this is not the case in the vertical direction in a gravity field. Pressure in a fluid increases with depth because more fluid rests on deeper layers, and the effect of this "extra weight" on a deeper layer is balanced by an increase in pressure (Fig. 3–5).

To obtain a relation for the variation of pressure with depth, consider a rectangular fluid element of height Δz, length Δx, and unit depth (into the page) in equilibrium, as shown in Fig. 3–6. Assuming the density of the fluid ρ to be constant, a force balance in the vertical z-direction gives

$$\sum F_z = ma_z = 0: \qquad P_2\,\Delta x - P_1\,\Delta x - \rho g\,\Delta x\,\Delta z = 0 \tag{3–6}$$

where $W = mg = \rho g\,\Delta x\,\Delta z$ is the weight of the fluid element. Dividing by Δx and rearranging gives

$$\Delta P = P_2 - P_1 = \rho g\,\Delta z = \gamma_s\,\Delta z \tag{3–7}$$

where $\gamma_s = \rho g$ is the *specific weight* of the fluid. Thus, we conclude that the pressure difference between two points in a constant density fluid is proportional to the vertical distance Δz between the points and the density ρ of the fluid. In other words, pressure in a fluid increases linearly with depth. This is what a diver experiences when diving deeper in a lake. For a given fluid, the vertical distance Δz is sometimes used as a measure of pressure, and it is called the *pressure head*.

We also conclude from Eq. 3–7 that for small to moderate distances, the variation of pressure with height is negligible for gases because of their low density. The pressure in a tank containing a gas, for example, can be considered to be uniform since the weight of the gas is too small to make a significant difference. Also, the pressure in a room filled with air can be assumed to be constant (Fig. 3–7).

If we take point 1 to be at the free surface of a liquid open to the atmosphere (Fig. 3–8), where the pressure is the atmospheric pressure P_{atm}, then the pressure at a depth h from the free surface becomes

$$P = P_{atm} + \rho g h \qquad \text{or} \qquad P_{gage} = \rho g h \tag{3–8}$$

Liquids are essentially incompressible substances, and thus the variation of density with depth is negligible. This is also the case for gases when the elevation change is not very large. The variation of density of liquids or gases with temperature can be significant, however, and may need to be considered when high accuracy is desired. Also, at great depths such as those encountered in oceans, the change in the density of a liquid can be significant because of the compression by the tremendous amount of liquid weight above.

The gravitational acceleration g varies from 9.807 m/s^2 at sea level to 9.764 m/s^2 at an elevation of 14,000 m where large passenger planes cruise. This is a change of just 0.4 percent in this extreme case. Therefore, g can be assumed to be constant with negligible error.

For fluids whose density changes significantly with elevation, a relation for the variation of pressure with elevation can be obtained by dividing Eq. 3–6 by $\Delta x\,\Delta z$, and taking the limit as $\Delta z \to 0$. It gives

$$\frac{dP}{dz} = -\rho g \tag{3–9}$$

The negative sign is due to our taking the positive z direction to be upward so that dP is negative when dz is positive since pressure decreases in an upward direction. When the variation of density with elevation is known,

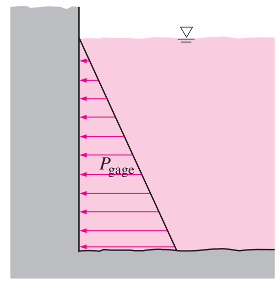


FIGURE 3–5
The pressure of a fluid at rest increases with depth (as a result of added weight).

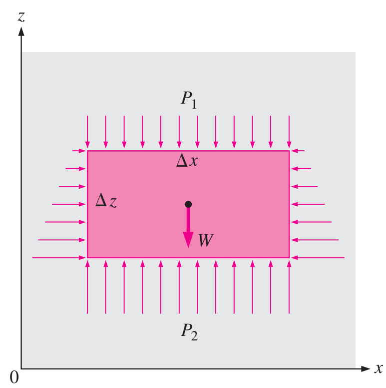


FIGURE 3–6
Free-body diagram of a rectangular fluid element in equilibrium.

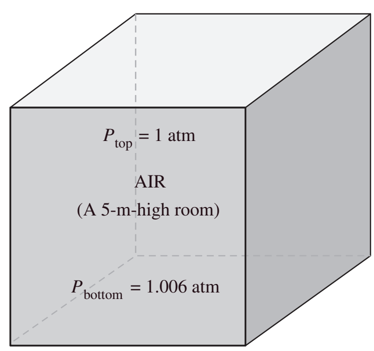


FIGURE 3–7
In a room filled with a gas, the variation of pressure with height is negligible.

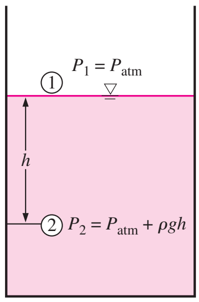


FIGURE 3–8
Pressure in a liquid at rest increases linearly with distance from the free surface.

the pressure difference between points 1 and 2 can be determined by integration to be

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g\, dz \qquad \text{(3–10)}$$

For constant density and constant gravitational acceleration, this relation reduces to Eq. 3–7, as expected.

Pressure in a fluid at rest is independent of the shape or cross section of the container. It changes with the vertical distance, but remains constant in other directions. Therefore, the pressure is the same at all points on a horizontal plane in a given fluid. The Dutch mathematician Simon Stevin (1548–1620) published in 1586 the principle illustrated in Fig. 3–9. Note that the pressures at points *A*, *B*, *C*, *D*, *E*, *F*, and *G* are the same since they are at the same depth, and they are interconnected by the same static fluid. However, the pressures at points *H* and *I* are not the same since these two points cannot be interconnected by the same fluid (i.e., we cannot draw a curve from point *I* to point *H* while remaining in the same fluid at all times), although they are at the same depth. (Can you tell at which point the pressure is higher?) Also, the pressure force exerted by the fluid is always normal to the surface at the specified points.

A consequence of the pressure in a fluid remaining constant in the horizontal direction is that *the pressure applied to a confined fluid increases the pressure throughout by the same amount*. This is called **Pascal's law,** after Blaise Pascal (1623–1662). Pascal also knew that the force applied by a fluid is proportional to the surface area. He realized that two hydraulic cylinders of different areas could be connected, and the larger could be used to exert a proportionally greater force than that applied to the smaller. "Pascal's machine" has been the source of many inventions that are a part of our daily lives such as hydraulic brakes and lifts. This is what enables us to lift

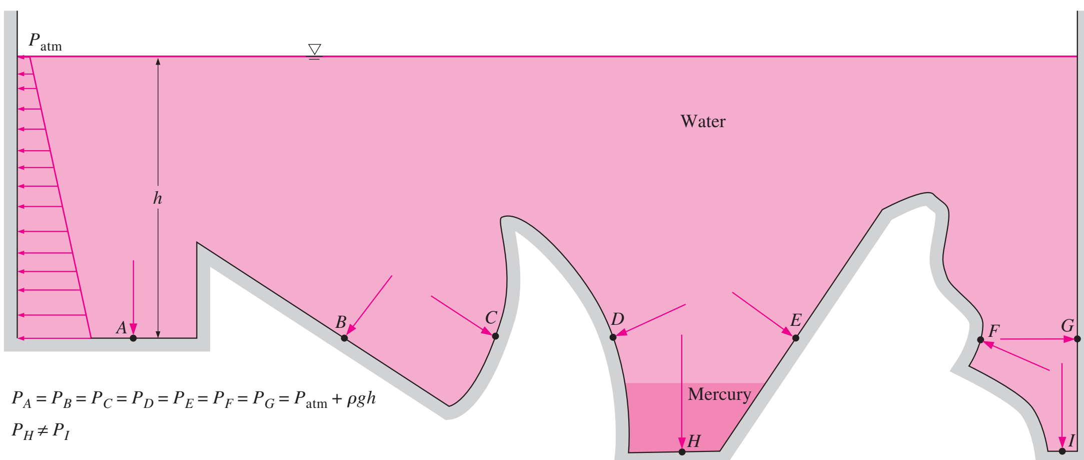


FIGURE 3–9
The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

a car easily by one arm, as shown in Fig. 3–10. Noting that $P_1 = P_2$ since both pistons are at the same level (the effect of small height differences is negligible, especially at high pressures), the ratio of output force to input force is determined to be

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1} \tag{3–11}$$

The area ratio A_2/A_1 is called the *ideal mechanical advantage* of the hydraulic lift. Using a hydraulic car jack with a piston area ratio of $A_2/A_1 = 10$, for example, a person can lift a 1000-kg car by applying a force of just 100 kgf (= 908 N).

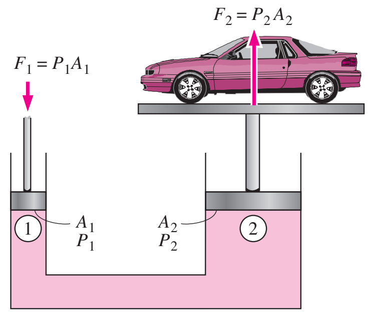


FIGURE 3–10
Lifting of a large weight by a small force by the application of Pascal's law.

3–2 ▪ THE MANOMETER

We notice from Eq. 3–7 that an elevation change of Δz in a fluid at rest corresponds to $\Delta P/\rho g$, which suggests that a fluid column can be used to measure pressure differences. A device based on this principle is called a **manometer,** and it is commonly used to measure small and moderate pressure differences. A manometer mainly consists of a glass or plastic U-tube containing one or more fluids such as mercury, water, alcohol, or oil. To keep the size of the manometer to a manageable level, heavy fluids such as mercury are used if large pressure differences are anticipated.

Consider the manometer shown in Fig. 3–11 that is used to measure the pressure in the tank. Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value. Furthermore, since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, $P_2 = P_1$.

The differential fluid column of height h is in static equilibrium, and it is open to the atmosphere. Then the pressure at point 2 is determined directly from Eq. 3–8 to be

$$P_2 = P_{atm} + \rho g h \tag{3–12}$$

where ρ is the density of the fluid in the tube. Note that the cross-sectional area of the tube has no effect on the differential height h, and thus the pressure exerted by the fluid. However, the diameter of the tube should be large enough (more than a few millimeters) to ensure that the surface tension effect and thus the capillary rise is negligible.

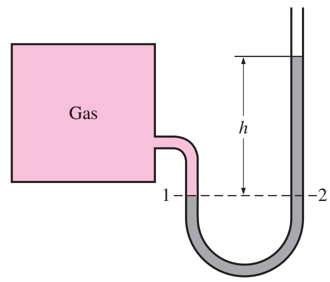


FIGURE 3–11
The basic manometer.

EXAMPLE 3–2 **Measuring Pressure with a Manometer**

A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown in Fig. 3–12. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

SOLUTION The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumptions The fluid in the tank is a gas whose density is much lower than the density of manometer fluid.

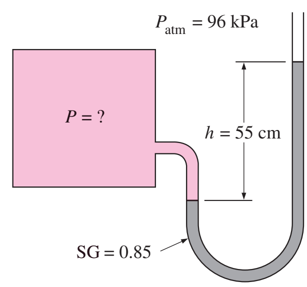


FIGURE 3–12
Schematic for Example 3–2.

Properties The specific gravity of the manometer fluid is given to be 0.85. We take the standard density of water to be 1000 kg/m³.

Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water, which is taken to be 1000 kg/m³:

$$\rho = \text{SG}\,(\rho_{H_2O}) = (0.85)(1000\text{ kg/m}^3) = 850\text{ kg/m}^3$$

Then from Eq. 3–12,

$$\begin{aligned} P &= P_{\text{atm}} + \rho g h \\ &= 96\text{ kPa} + (850\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.55\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) \\ &= \mathbf{100.6\text{ kPa}} \end{aligned}$$

Discussion Note that the gage pressure in the tank is 4.6 kPa.

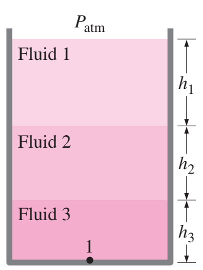


FIGURE 3–13
In stacked-up fluid layers, the pressure change across a fluid layer of density ρ and height h is ρgh.

Many engineering problems and some manometers involve multiple immiscible fluids of different densities stacked on top of each other. Such systems can be analyzed easily by remembering that (1) the pressure change across a fluid column of height h is $\Delta P = \rho gh$, (2) pressure increases downward in a given fluid and decreases upward (i.e., $P_{\text{bottom}} > P_{\text{top}}$), and (3) two points at the same elevation in a continuous fluid at rest are at the same pressure.

The last principle, which is a result of *Pascal's law,* allows us to "jump" from one fluid column to the next in manometers without worrying about pressure change as long as we don't jump over a different fluid, and the fluid is at rest. Then the pressure at any point can be determined by starting with a point of known pressure and adding or subtracting ρgh terms as we advance toward the point of interest. For example, the pressure at the bottom of the tank in Fig. 3–13 can be determined by starting at the free surface where the pressure is P_{atm}, moving downward until we reach point 1 at the bottom, and setting the result equal to P_1. It gives

$$P_{\text{atm}} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

In the special case of all fluids having the same density, this relation reduces to Eq. 3–12, as expected.

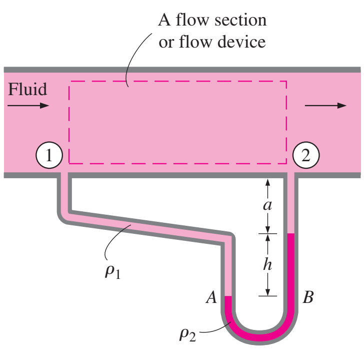


FIGURE 3–14
Measuring the pressure drop across a flow section or a flow device by a differential manometer.

Manometers are particularly well-suited to measure pressure drops across a horizontal flow section between two specified points due to the presence of a device such as a valve or heat exchanger or any resistance to flow. This is done by connecting the two legs of the manometer to these two points, as shown in Fig. 3–14. The working fluid can be either a gas or a liquid whose density is ρ_1. The density of the manometer fluid is ρ_2, and the differential fluid height is h.

A relation for the pressure difference $P_1 - P_2$ can be obtained by starting at point 1 with P_1, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_2:

$$P_1 + \rho_1 g(a + h) - \rho_2 gh - \rho_1 ga = P_2 \tag{3–13}$$

Note that we jumped from point A horizontally to point B and ignored the part underneath since the pressure at both points is the same. Simplifying,

$$P_1 - P_2 = (\rho_2 - \rho_1)gh \tag{3–14}$$

Note that the distance *a* has no effect on the result, but must be included in the analysis. Also, when the fluid flowing in the pipe is a gas, then $\rho_1 \ll \rho_2$ and the relation in Eq. 3–14 simplifies to $P_1 - P_2 \approx \rho_2 gh$.

EXAMPLE 3–3 Measuring Pressure with a Multifluid Manometer

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. 3–15. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

SOLUTION The pressure in a pressurized water tank is measured by a multifluid manometer. The air pressure in the tank is to be determined.
Assumption The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air–water interface.
Properties The densities of water, oil, and mercury are given to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.
Analysis Starting with the pressure at point 1 at the air–water interface, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$$P_1 + \rho_{water} gh_1 + \rho_{oil} gh_2 - \rho_{mercury} gh_3 = P_{atm}$$

Solving for P_1 and substituting,

$$\begin{aligned} P_1 &= P_{atm} - \rho_{water} gh_1 - \rho_{oil} gh_2 + \rho_{mercury} gh_3 \\ &= P_{atm} + g(\rho_{mercury} h_3 - \rho_{water} h_1 - \rho_{oil} h_2) \\ &= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (850 \text{ kg/m}^3)(0.2 \text{ m})]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= \mathbf{130 \text{ kPa}} \end{aligned}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis considerably. Also note that mercury is a toxic fluid, and mercury manometers and thermometers are being replaced by ones with safer fluids because of the risk of exposure to mercury vapor during an accident.

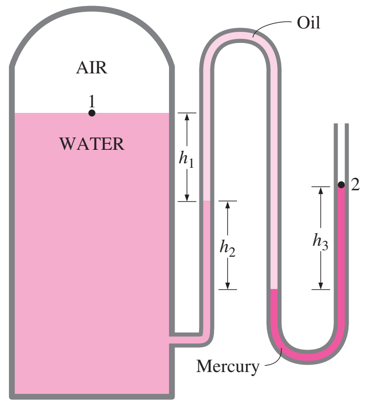


FIGURE 3–15
Schematic for Example 3–3; drawing not to scale.

EXAMPLE 3–4 Analyzing a Multifluid Manometer with EES

Reconsider the multifluid manometer discussed in Example 3–3. Determine the air pressure in the tank using EES. Also determine what the differential fluid height h_3 would be for the same air pressure if the mercury in the last column were replaced by seawater with a density of 1030 kg/m^3.

SOLUTION The pressure in a water tank is measured by a multifluid manometer. The air pressure in the tank and the differential fluid height h_3 if mercury is replaced by seawater are to be determined using EES.

Analysis We start the EES program by double-clicking on its icon, open a new file, and type the following on the blank screen that appears (we express the atmospheric pressure in Pa for unit consistency):

g=9.81

Patm=85600

h1=0.1; h2=0.2; h3=0.35

rw=1000; roil=850; rm=13600

P1+rw*g*h1+roil*g*h2−rm*g*h3=Patm

Here P1 is the only unknown, and it is determined by EES to be

$$P_1 = 129647 \text{ Pa} \cong \mathbf{130\ kPa}$$

which is identical to the result obtained in Example 3–3. The height of the fluid column h_3 when mercury is replaced by seawater is determined easily by replacing "h3=0.35" by "P1=129647" and "rm=13600" by "rm=1030," and clicking on the calculator symbol. It gives

$$h_3 = \mathbf{4.62\ m}$$

Discussion Note that we used the screen like a paper pad and wrote down the relevant information together with the applicable relations in an organized manner. EES did the rest. Equations can be written on separate lines or on the same line by separating them by semicolons, and blank or comment lines can be inserted for readability. EES makes it very easy to ask "what if" questions and to perform parametric studies, as explained in Appendix 3 on the DVD.

Other Pressure Measurement Devices

Another type of commonly used mechanical pressure measurement device is the **Bourdon tube,** named after the French engineer and inventor Eugene Bourdon (1808–1884), which consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle (Fig. 3–16). When the tube is open to the atmosphere, the tube is undeflected, and the needle on the dial at this state is calibrated to read zero (gage pressure). When the fluid inside the tube is pressurized, the tube stretches and moves the needle in proportion to the pressure applied.

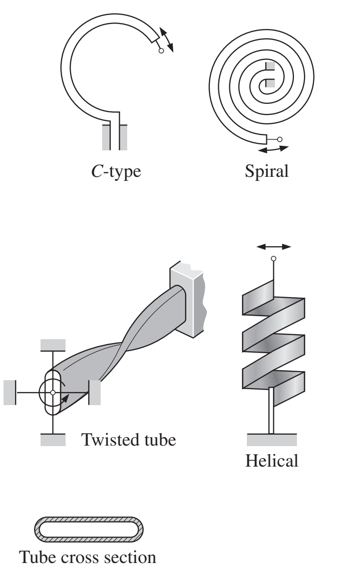


FIGURE 3–16
Various types of Bourdon tubes used to measure pressure.

Electronics have made their way into every aspect of life, including pressure measurement devices. Modern pressure sensors, called **pressure transducers,** use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance. Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts. They can measure pressures from less than a millionth of 1 atm to several thousands of atm.

A wide variety of pressure transducers is available to measure gage, absolute, and differential pressures in a wide range of applications. *Gage pressure transducers* use the atmospheric pressure as a reference by venting the back side of the pressure-sensing diaphragm to the atmosphere, and they give a zero signal output at atmospheric pressure regardless of altitude. The *absolute pressure transducers* are calibrated to have a zero signal output at full vacuum. *Differential pressure transducers* measure the pressure difference between two locations directly instead of using two pressure transducers and taking their difference.

Strain-gage pressure transducers work by having a diaphragm deflect between two chambers open to the pressure inputs. As the diaphragm stretches in response to a change in pressure difference across it, the strain gage stretches and a Wheatstone bridge circuit amplifies the output. A capacitance transducer works similarly, but capacitance change is measured instead of resistance change as the diaphragm stretches.

Piezoelectric transducers, also called solid-state pressure transducers, work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure. This phenomenon, first discovered by brothers Pierre and Jacques Curie in 1880, is called the piezoelectric (or press-electric) effect. Piezoelectric pressure transducers have a much faster frequency response compared to the diaphragm units and are very suitable for high-pressure applications, but they are generally not as sensitive as the diaphragm-type transducers.

3–3 ▪ THE BAROMETER AND ATMOSPHERIC PRESSURE

Atmospheric pressure is measured by a device called a **barometer;** thus, the atmospheric pressure is often referred to as the *barometric pressure*.

The Italian Evangelista Torricelli (1608–1647) was the first to conclusively prove that the atmospheric pressure can be measured by inverting a mercury-filled tube into a mercury container that is open to the atmosphere, as shown in Fig. 3–17. The pressure at point B is equal to the atmospheric pressure, and the pressure at C can be taken to be zero since there is only mercury vapor above point C and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation. Writing a force balance in the vertical direction gives

$$P_{atm} = \rho g h \qquad \textbf{(3–15)}$$

where ρ is the density of mercury, g is the local gravitational acceleration, and h is the height of the mercury column above the free surface. Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer (Fig. 3–18).

A frequently used pressure unit is the *standard atmosphere,* which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg} = 13,595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2). If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3 m would be needed. Pressure is sometimes expressed (especially by weather forecasters) in terms of the height of the mercury column. The standard atmospheric pressure, for example, is 760 mmHg (29.92 inHg) at 0°C. The unit mmHg is also called the **torr** in honor of Torricelli. Therefore, 1 atm = 760 torr and 1 torr = 133.3 Pa.

The standard atmospheric pressure P_{atm} changes from 101.325 kPa at sea level to 89.88, 79.50, 54.05, 26.5, and 5.53 kPa at altitudes of 1000, 2000, 5000, 10,000, and 20,000 meters, respectively. The standard atmospheric pressure in Denver (elevation = 1610 m), for example, is 83.4 kPa.

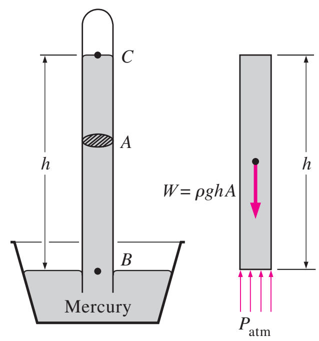


FIGURE 3–17
The basic barometer.

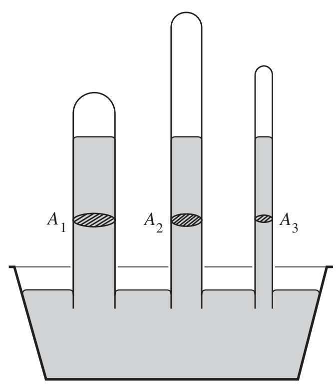


FIGURE 3–18
The length or the cross-sectional area of the tube has no effect on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effects.

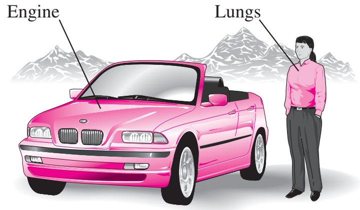


FIGURE 3–19
At high altitudes, a car engine generates less power and a person gets less oxygen because of the lower density of air.

Remember that the atmospheric pressure at a location is simply the weight of the air above that location per unit surface area. Therefore, it changes not only with elevation but also with weather conditions.

The decline of atmospheric pressure with elevation has far-reaching ramifications in daily life. For example, cooking takes longer at high altitudes since water boils at a lower temperature at lower atmospheric pressures. Nose bleeding is a common experience at high altitudes since the difference between the blood pressure and the atmospheric pressure is larger in this case, and the delicate walls of veins in the nose are often unable to withstand this extra stress.

For a given temperature, the density of air is lower at high altitudes, and thus a given volume contains less air and less oxygen. So it is no surprise that we tire more easily and experience breathing problems at high altitudes. To compensate for this effect, people living at higher altitudes develop more efficient lungs. Similarly, a 2.0-L car engine will act like a 1.7-L car engine at 1500 m altitude (unless it is turbocharged) because of the 15 percent drop in pressure and thus 15 percent drop in the density of air (Fig. 3–19). A fan or compressor will displace 15 percent less air at that altitude for the same volume displacement rate. Therefore, larger cooling fans may need to be selected for operation at high altitudes to ensure the specified mass flow rate. The lower pressure and thus lower density also affects lift and drag: airplanes need a longer runway at high altitudes to develop the required lift, and they climb to very high altitudes for cruising for reduced drag and thus better fuel efficiency.

EXAMPLE 3–5 Measuring Atmospheric Pressure with a Barometer

Determine the atmospheric pressure at a location where the barometric reading is 740 mm Hg and the gravitational acceleration is $g = 9.81\ \text{m/s}^2$. Assume the temperature of mercury to be 10°C, at which its density is 13,570 kg/m^3.

SOLUTION The barometric reading at a location in height of mercury column is given. The atmospheric pressure is to be determined.
Assumptions The temperature of mercury is assumed to be 10°C.
Properties The density of mercury is given to be 13,570 kg/m^3.
Analysis From Eq. 3–15, the atmospheric pressure is determined to be

$$\begin{aligned} P_{\text{atm}} &= \rho g h \\ &= (13{,}570\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.74\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{98.5\ kPa} \end{aligned}$$

Discussion Note that density changes with temperature, and thus this effect should be considered in calculations.

EXAMPLE 3–6 Effect of Piston Weight on Pressure in a Cylinder

The piston of a vertical piston–cylinder device containing a gas has a mass of 60 kg and a cross-sectional area of 0.04 m^2, as shown in Fig. 3–20. The local atmospheric pressure is 0.97 bar, and the gravitational acceleration is 9.81 m/s^2. (*a*) Determine the pressure inside the cylinder. (*b*) If some heat is transferred to the gas and its volume is doubled, do you expect the pressure inside the cylinder to change?

SOLUTION A gas is contained in a vertical cylinder with a heavy piston. The pressure inside the cylinder and the effect of volume change on pressure are to be determined.

Assumptions Friction between the piston and the cylinder is negligible.

Analysis (*a*) The gas pressure in the piston–cylinder device depends on the atmospheric pressure and the weight of the piston. Drawing the free-body diagram of the piston as shown in Fig. 3–20 and balancing the vertical forces yield

$$PA = P_{atm}A + W$$

Solving for P and substituting,

$$\begin{aligned} P &= P_{atm} + \frac{mg}{A} \\ &= 0.97 \text{ bar} + \frac{(60 \text{ kg})(9.81 \text{ m/s}^2)}{0.04 \text{ m}^2}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right) \\ &= \mathbf{1.12 \text{ bars}} \end{aligned}$$

(*b*) The volume change will have no effect on the free-body diagram drawn in part (*a*), and therefore the pressure inside the cylinder will remain the same.

Discussion If the gas behaves as an ideal gas, the absolute temperature doubles when the volume is doubled at constant pressure.

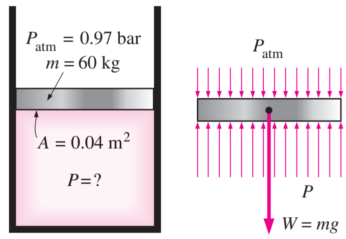


FIGURE 3–20
Schematic for Example 3–6, and the free-body diagram of the piston.

EXAMPLE 3–7 Hydrostatic Pressure in a Solar Pond with Variable Density

Solar ponds are small artificial lakes of a few meters deep that are used to store solar energy. The rise of heated (and thus less dense) water to the surface is prevented by adding salt at the pond bottom. In a typical salt gradient solar pond, the density of water increases in the gradient zone, as shown in Fig. 3–21, and the density can be expressed as

$$\rho = \rho_0\sqrt{1 + \tan^2\left(\frac{\pi}{4}\frac{z}{H}\right)}$$

where ρ_0 is the density on the water surface, z is the vertical distance measured downward from the top of the gradient zone, and H is the thickness of

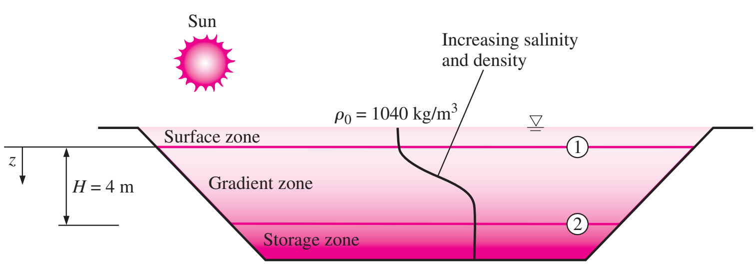


FIGURE 3–21
Schematic for Example 3–7.

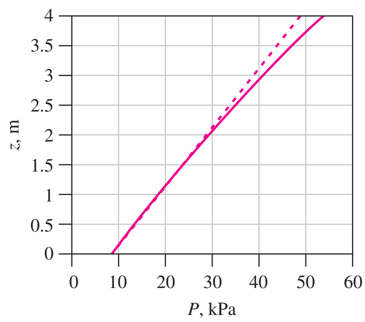


FIGURE 3–22
The variation of gage pressure with depth in the gradient zone of the solar pond.

the gradient zone. For $H = 4$ m, $\rho_0 = 1040$ kg/m^3, and a thickness of 0.8 m for the surface zone, calculate the gage pressure at the bottom of the gradient zone.

SOLUTION The variation of density of saline water in the gradient zone of a solar pond with depth is given. The gage pressure at the bottom of the gradient zone is to be determined.

Assumptions The density in the surface zone of the pond is constant.

Properties The density of brine on the surface is given to be 1040 kg/m^3.

Analysis We label the top and the bottom of the gradient zone as 1 and 2, respectively. Noting that the density of the surface zone is constant, the gage pressure at the bottom of the surface zone (which is the top of the gradient zone) is

$$P_1 = \rho g h_1 = (1040 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 8.16 \text{ kPa}$$

since 1 kN/m^2 = 1 kPa. The differential change in hydrostatic pressure across a vertical distance of dz is given by

$$dP = \rho g\, dz$$

Integrating from the top of the gradient zone (point 1 where $z = 0$) to any location z in the gradient zone (no subscript) gives

$$P - P_1 = \int_0^z \rho g\, dz \qquad \rightarrow \qquad P = P_1 + \int_0^z \rho_0 \sqrt{1 + \tan^2\left(\frac{\pi}{4}\frac{z}{H}\right)}\, g\, dz$$

Performing the integration gives the variation of gage pressure in the gradient zone to be

$$P = P_1 + \rho_0 g \frac{4H}{\pi} \sinh^{-1}\left(\tan \frac{\pi}{4}\frac{z}{H}\right)$$

Then the pressure at the bottom of the gradient zone ($z = H = 4$ m) becomes

$$P_2 = 8.16 \text{ kPa} + (1040 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\frac{4(4 \text{ m})}{\pi} \sinh^{-1}\left(\tan \frac{\pi}{4}\frac{4}{4}\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= \mathbf{54.0 \text{ kPa (gage)}}$$

Discussion The variation of gage pressure in the gradient zone with depth is plotted in Fig. 3–22. The dashed line indicates the hydrostatic pressure for the case of constant density at 1040 kg/m^3 and is given for reference. Note that the variation of pressure with depth is not linear when density varies with depth.

3–4 ▪ INTRODUCTION TO FLUID STATICS

Fluid statics deals with problems associated with fluids at rest. The fluid can be either gaseous or liquid. Fluid statics is generally referred to as *hydrostatics* when the fluid is a liquid and as *aerostatics* when the fluid is a gas. In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the *normal stress,* which is the pressure, and the variation of pressure is due only to the weight of the fluid. Therefore, the topic of fluid statics has significance only in gravity fields, and the force relations developed naturally involve the gravitational acceleration g. The force exerted on a surface by a fluid at rest is normal to the surface at the point of contact since there is no relative motion between the fluid and the solid surface, and thus no shear forces can act parallel to the surface.

Fluid statics is used to determine the forces acting on floating or submerged bodies and the forces developed by devices like hydraulic presses and car jacks. The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics. The complete description of the resultant hydrostatic force acting on a submerged surface requires the determination of the magnitude, the direction, and the line of action of the force. In Sections 3–5 and 3–6, we consider the forces acting on both plane and curved surfaces of submerged bodies due to pressure.

FIGURE 3–23
Hoover Dam.
Courtesy United States Department of the Interior, Bureau of Reclamation-Lower Colorado Region.

3–5 ▪ HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

A plate exposed to a liquid, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface (Fig. 3–23). On a *plane* surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the *magnitude* of the force and its *point of application,* which is called the **center of pressure.** In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant. In such cases, it is convenient to subtract atmospheric pressure and work with the gage pressure only (Fig. 3–24). For example, $P_{gage} = \rho gh$ at the bottom of the lake.

Consider the top surface of a flat plate of arbitrary shape completely submerged in a liquid, as shown in Fig. 3–25 together with its top view. The plane of this surface (normal to the page) intersects the horizontal free surface with an angle θ, and we take the line of intersection to be the x-axis. The absolute pressure above the liquid is P_0, which is the local atmospheric pressure P_{atm} if the liquid is open to the atmosphere (but P_0 may be different

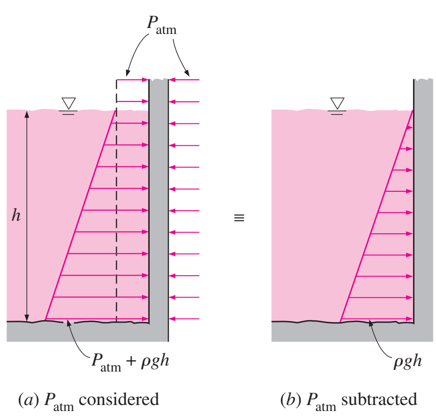


FIGURE 3–24
When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

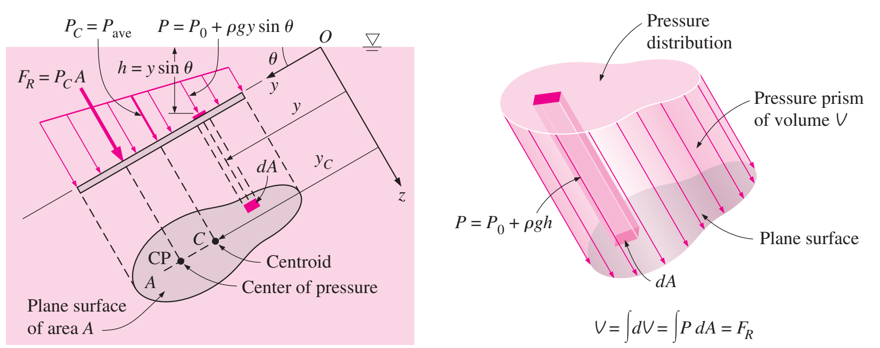


FIGURE 3–25
Hydrostatic force on an inclined plane surface completely submerged in a liquid.

than P_{atm} if the space above the liquid is evacuated or pressurized). Then the absolute pressure at any point on the plate is

$$P = P_0 + \rho g h = P_0 + \rho g y \sin\theta \qquad \textbf{(3–16)}$$

where h is the vertical distance of the point from the free surface and y is the distance of the point from the x-axis (from point O in Fig. 3–25). The resultant hydrostatic force F_R acting on the surface is determined by integrating the force $P\,dA$ acting on a differential area dA over the entire surface area,

$$F_R = \int_A P\,dA = \int_A (P_0 + \rho g y \sin\theta)\,dA = P_0 A + \rho g \sin\theta \int_A y\,dA \qquad \textbf{(3–17)}$$

But the *first moment of area* $\int_A y\,dA$ is related to the y-coordinate of the centroid (or center) of the surface by

$$y_C = \frac{1}{A}\int_A y\,dA \qquad \textbf{(3–18)}$$

Substituting,

$$F_R = (P_0 + \rho g y_C \sin\theta)A = (P_0 + \rho g h_C)A = P_C A = P_{ave} A \qquad \textbf{(3–19)}$$

where $P_C = P_0 + \rho g h_C$ is the pressure at the centroid of the surface, which is equivalent to the *average* pressure on the surface, and $h_C = y_C \sin\theta$ is the *vertical distance* of the centroid from the free surface of the liquid (Fig. 3–26). Thus we conclude that:

> The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface (Fig. 3–27).

The pressure P_0 is usually atmospheric pressure, which can be ignored in most cases since it acts on both sides of the plate. When this is not the case, a practical way of accounting for the contribution of P_0 to the resultant

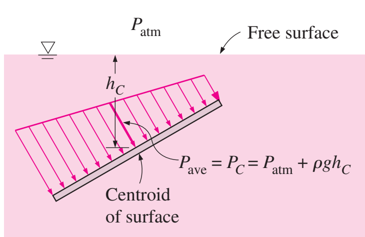


FIGURE 3–26
The pressure at the centroid of a surface is equivalent to the *average* pressure on the surface.

force is simply to add an equivalent depth $h_{\text{equiv}} = P_0/\rho g$ to h_C; that is, to assume the presence of an additional liquid layer of thickness h_{equiv} on top of the liquid with absolute vacuum above.

Next we need to determine the line of action of the resultant force F_R. Two parallel force systems are equivalent if they have the same magnitude and the same moment about any point. The line of action of the resultant hydrostatic force, in general, does not pass through the centroid of the surface—it lies underneath where the pressure is higher. The point of intersection of the line of action of the resultant force and the surface is the **center of pressure.** The vertical location of the line of action is determined by equating the moment of the resultant force to the moment of the distributed pressure force about the x-axis. It gives

$$y_P F_R = \int_A yP\,dA = \int_A y(P_0 + \rho g y \sin\theta)\,dA = P_0 \int_A y\,dA + \rho g \sin\theta \int_A y^2\,dA$$

or

$$y_P F_R = P_0 y_C A + \rho g \sin\theta\, I_{xx,\,O} \tag{3–20}$$

where y_P is the distance of the center of pressure from the x-axis (point O in Fig. 3–27) and $I_{xx,\,O} = \int_A y^2\,dA$ is the *second moment of area* (also called the *area moment of inertia*) about the x-axis. The second moments of area are widely available for common shapes in engineering handbooks, but they are usually given about the axes passing through the centroid of the area. Fortunately, the second moments of area about two parallel axes are related to each other by the *parallel axis theorem,* which in this case is expressed as

$$I_{xx,\,O} = I_{xx,\,C} + y_C^2 A \tag{3–21}$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area and y_C (the y-coordinate of the centroid) is the distance between the two parallel axes. Substituting the F_R relation from Eq. 3–19 and the $I_{xx,\,O}$ relation from Eq. 3–21 into Eq. 3–20 and solving for y_P gives

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A} \tag{3–22a}$$

For $P_0 = 0$, which is usually the case when the atmospheric pressure is ignored, it simplifies to

$$y_P = y_C + \frac{I_{xx,\,C}}{y_C A} \tag{3–22b}$$

Knowing y_P, the vertical distance of the center of pressure from the free surface is determined from $h_P = y_P \sin\theta$.

The $I_{xx,\,C}$ values for some common areas are given in Fig. 3–28. For these and other areas that possess symmetry about the y-axis, the center of pressure lies on the y-axis directly below the centroid. The location of the center of pressure in such cases is simply the point on the surface of the vertical plane of symmetry at a distance h_P from the free surface.

Pressure acts normal to the surface, and the hydrostatic forces acting on a flat plate of any shape form a volume whose base is the plate area and

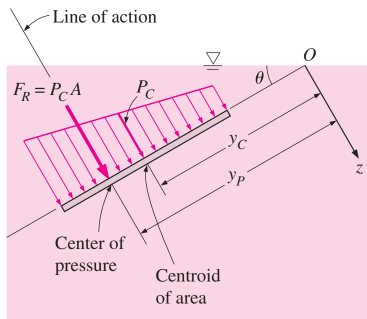


FIGURE 3–27
The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure.

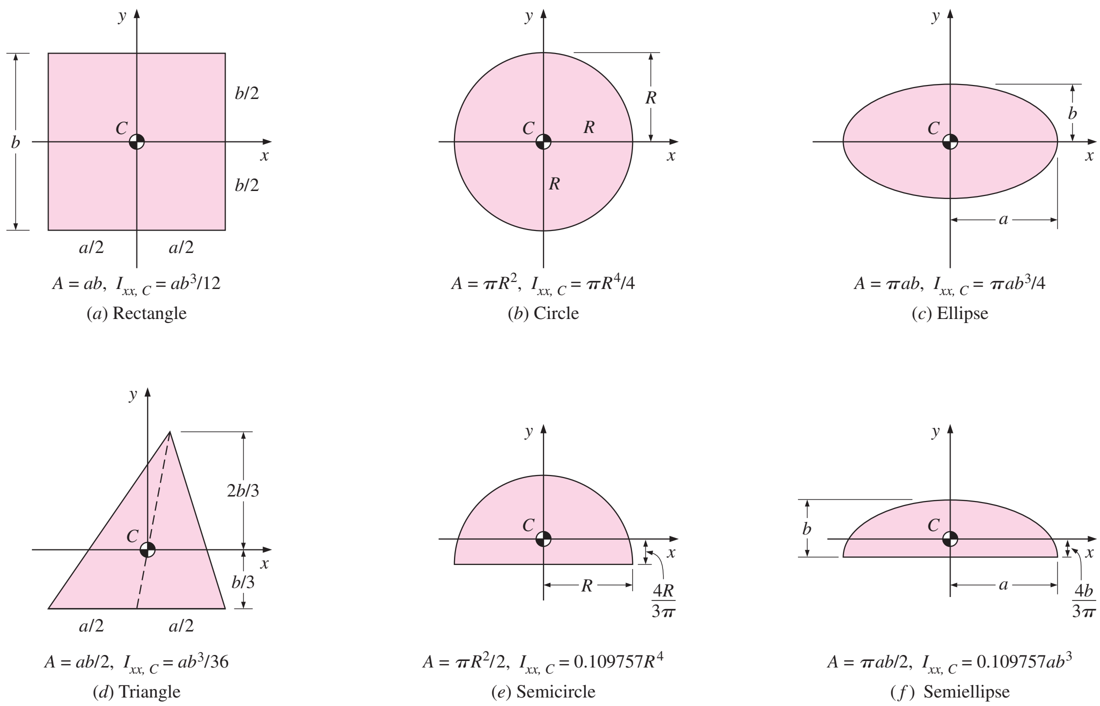


FIGURE 3–28
The centroid and the centroidal moments of inertia for some common geometries.

whose height is the linearly varying pressure, as shown in Fig. 3–29. This virtual **pressure prism** has an interesting physical interpretation: its *volume* is equal to the *magnitude* of the resultant hydrostatic force acting on the plate since $V = \int P\,dA$, and the line of action of this force passes through the *centroid* of this homogeneous prism. The projection of the centroid on the plate is the *pressure center.* Therefore, with the concept of pressure prism, the problem of describing the resultant hydrostatic force on a plane surface reduces to finding the volume and the two coordinates of the centroid of this pressure prism.

Special Case: Submerged Rectangular Plate

Consider a completely submerged rectangular flat plate of height b and width a tilted at an angle θ from the horizontal and whose top edge is horizontal and is at a distance s from the free surface along the plane of the plate, as shown in Fig. 3–30*a*. The resultant hydrostatic force on the upper surface is equal to the average pressure, which is the pressure at the midpoint of the surface, times the surface area A. That is,

Tilted rectangular plate: $$F_R = P_C A = [P_0 + \rho g(s + b/2)\sin\theta]ab \qquad \textbf{(3–23)}$$

The force acts at a vertical distance of $h_P = y_P \sin\theta$ from the free surface directly beneath the centroid of the plate where, from Eq. 3–22a,

$$y_P = s + \frac{b}{2} + \frac{ab^3/12}{[s + b/2 + P_0/(\rho g \sin\theta)]ab}$$

$$= s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin\theta)]} \tag{3–24}$$

When the upper edge of the plate is at the free surface and thus $s = 0$, Eq. 3–23 reduces to

Tilted rectangular plate ($s = 0$): $$F_R = [P_0 + \rho g(b \sin\theta)/2]ab \tag{3–25}$$

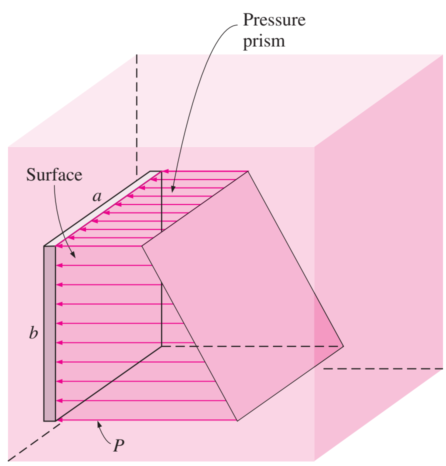
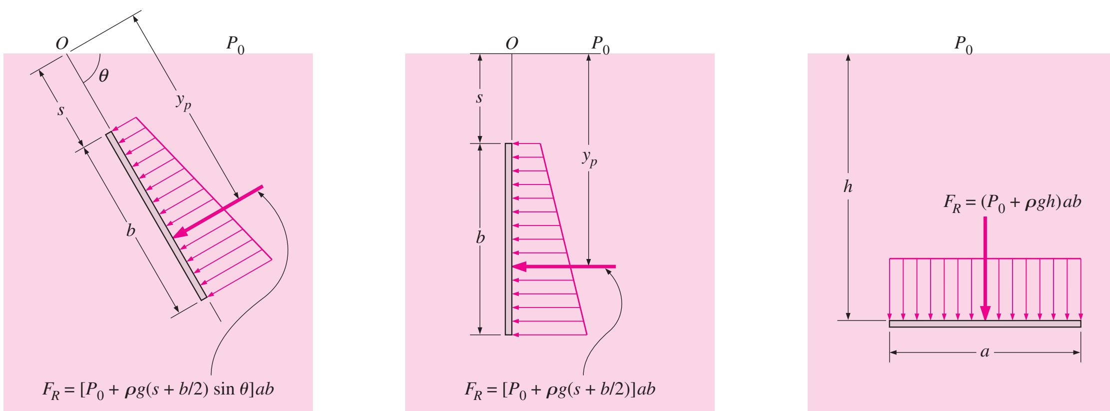


FIGURE 3–29

The hydrostatic forces acting on a plane surface form a volume whose base (left face) is the surface and whose height is the pressure.



(*a*) Tilted plate (*b*) Vertical plate (*c*) Horizontal plate

FIGURE 3–30

Hydrostatic force acting on the top surface of a submerged rectangular plate for tilted, vertical, and horizontal cases.

For a completely submerged *vertical* plate ($\theta = 90°$) whose top edge is horizontal, the hydrostatic force can be obtained by setting $\sin\theta = 1$ (Fig. 3–30*b*)

Vertical rectangular plate: $$F_R = [P_0 + \rho g(s + b/2)]ab \quad \textbf{(3–26)}$$

Vertical rectangular plate ($s = 0$): $$F_R = (P_0 + \rho gb/2)ab \quad \textbf{(3–27)}$$

When the effect of P_0 is ignored since it acts on both sides of the plate, the hydrostatic force on a vertical rectangular surface of height b whose top edge is horizontal and at the free surface is $F_R = \rho gab^2/2$ acting at a distance of $2b/3$ from the free surface directly beneath the centroid of the plate.

The pressure distribution on a submerged *horizontal* surface is uniform, and its magnitude is $P = P_0 + \rho gh$, where h is the distance of the surface from the free surface. Therefore, the hydrostatic force acting on a horizontal rectangular surface is

Horizontal rectangular plate: $$F_R = (P_0 + \rho gh)ab \quad \textbf{(3–28)}$$

and it acts through the midpoint of the plate (Fig. 3–30*c*).

EXAMPLE 3–8 Hydrostatic Force Acting on the Door of a Submerged Car

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 3–31). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

SOLUTION A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed.

Assumptions **1** The bottom surface of the lake is horizontal. **2** The passenger cabin is well-sealed so that no water leaks inside. **3** The door can be approximated as a vertical rectangular plate. **4** The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in, and

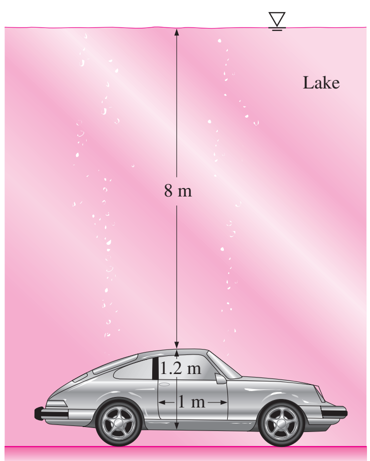


FIGURE 3–31
Schematic for Example 3–8.

thus no compression of the air inside. Therefore, atmospheric pressure cancels out in the calculations since it acts on both sides of the door. **5** The weight of the car is larger than the buoyant force acting on it.

Properties We take the density of lake water to be 1000 kg/m³ throughout.

Analysis The average pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

$$P_{\text{ave}} = P_C = \rho g h_C = \rho g(s + b/2)$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 + 1.2/2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= \mathbf{84.4\ kN/m^2}$$

Then the resultant hydrostatic force on the door becomes

$$F_R = P_{\text{ave}} A = (84.4 \text{ kN/m}^2)(1 \text{ m} \times 1.2 \text{ m}) = \mathbf{101.3\ kN}$$

The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from Eq. 3–24 by setting $P_0 = 0$ to be

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.2}{2} + \frac{1.2^2}{12(8 + 1.2/2)} = \mathbf{8.61\ m}$$

Discussion A strong person can lift 100 kg, whose weight is 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN · m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN · m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

3–6 ▪ HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES

For a submerged curved surface, the determination of the resultant hydrostatic force is more involved since it typically requires the integration of the pressure forces that change direction along the curved surface. The concept of the pressure prism in this case is not much help either because of the complicated shapes involved.

The easiest way to determine the resultant hydrostatic force F_R acting on a two-dimensional curved surface is to determine the horizontal and vertical components F_H and F_V separately. This is done by considering the free-body diagram of the liquid block enclosed by the curved surface and the two plane surfaces (one horizontal and one vertical) passing through the two ends of the curved surface, as shown in Fig. 3–32. Note that the vertical surface of the liquid block considered is simply the projection of the curved surface on a *vertical plane,* and the horizontal surface is the projection of the curved surface on a *horizontal plane.* The resultant force acting on the

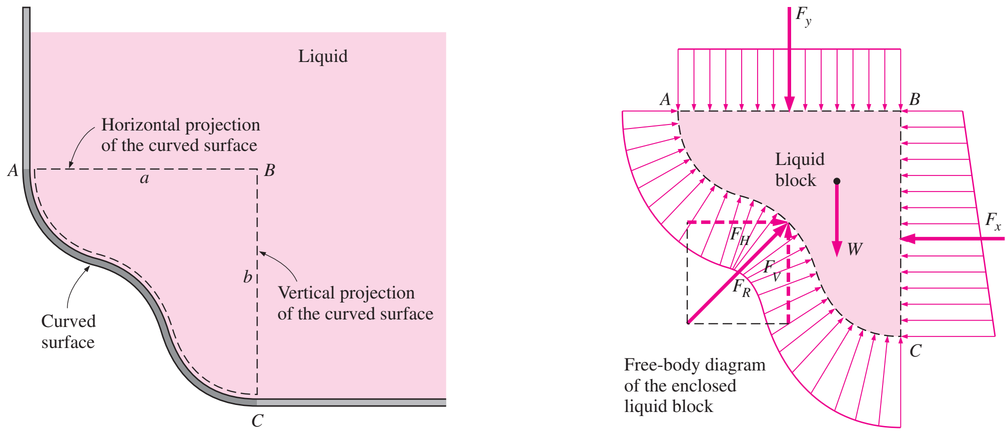


FIGURE 3–32
Determination of the hydrostatic force acting on a submerged curved surface.

curved solid surface is then equal and opposite to the force acting on the curved liquid surface (Newton's third law).

The force acting on the imaginary horizontal or vertical plane surface and its line of action can be determined as discussed in Section 3–5. The weight of the enclosed liquid block of volume V is simply $W = \rho g V$, and it acts downward through the centroid of this volume. Noting that the fluid block is in static equilibrium, the force balances in the horizontal and vertical directions give

Horizontal force component on curved surface: $$F_H = F_x \tag{3–29}$$

Vertical force component on curved surface: $$F_V = F_y + W \tag{3–30}$$

where the summation $F_y + W$ is a vector addition (i.e., add magnitudes if both act in the same direction and subtract if they act in opposite directions). Thus, we conclude that

1. The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.
2. The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

The magnitude of the resultant hydrostatic force acting on the curved surface is $F_R = \sqrt{F_H^2 + F_V^2}$, and the tangent of the angle it makes with the horizontal is $\tan \alpha = F_V/F_H$. The exact location of the line of action of the resultant force (e.g., its distance from one of the end points of the curved surface) can be determined by taking a moment about an appropriate point. These discussions are valid for all curved surfaces regardless of whether they are above or below the liquid. Note that in the case of a *curved surface above a liquid,* the weight of the liquid is *subtracted* from the vertical component of the hydrostatic force since they act in opposite directions (Fig. 3–33).

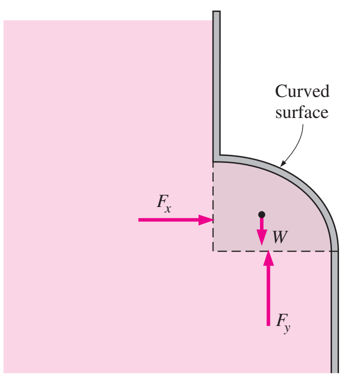


FIGURE 3–33
When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.

When the curved surface is a *circular arc* (full circle or any part of it), the resultant hydrostatic force acting on the surface always passes through the center of the circle. This is because the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus, the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point (Fig. 3–34).

Finally, hydrostatic forces acting on a plane or curved surface submerged in a **multilayered fluid** of different densities can be determined by considering different parts of surfaces in different fluids as different surfaces, finding the force on each part, and then adding them using vector addition. For a plane surface, it can be expressed as (Fig. 3–35)

Plane surface in a multilayered fluid: $$F_R = \sum F_{R,i} = \sum P_{C,i} A_i \qquad \text{(3–31)}$$

where $P_{C,i} = P_0 + \rho_i g h_{C,i}$ is the pressure at the centroid of the portion of the surface in fluid i and A_i is the area of the plate in that fluid. The line of action of this equivalent force can be determined from the requirement that the moment of the equivalent force about any point is equal to the sum of the moments of the individual forces about the same point.

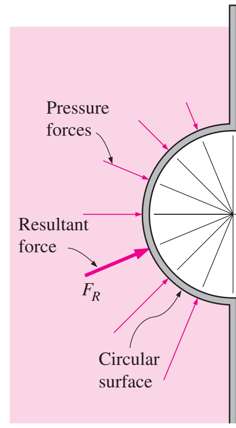


FIGURE 3–34
The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.

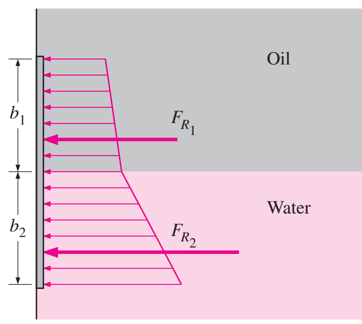


FIGURE 3–35
The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.

EXAMPLE 3–9 A Gravity-Controlled Cylindrical Gate

A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate, as shown in Fig. 3–36. When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per m length of the cylinder.

SOLUTION The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per m length are to be determined.

Assumptions **1** Friction at the hinge is negligible. **2** Atmospheric pressure acts on both sides of the gate, and thus it cancels out.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$F_H = F_x = P_{\text{ave}} A = \rho g h_C A = \rho g (s + R/2) A$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4.2 + 0.8/2 \text{ m})(0.8 \text{ m} \times 1 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{36.1\ kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{\text{ave}} A = \rho g h_C A = \rho g h_{\text{bottom}} A$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})(0.8 \text{ m} \times 1 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$
$$= 39.2 \text{ kN}$$

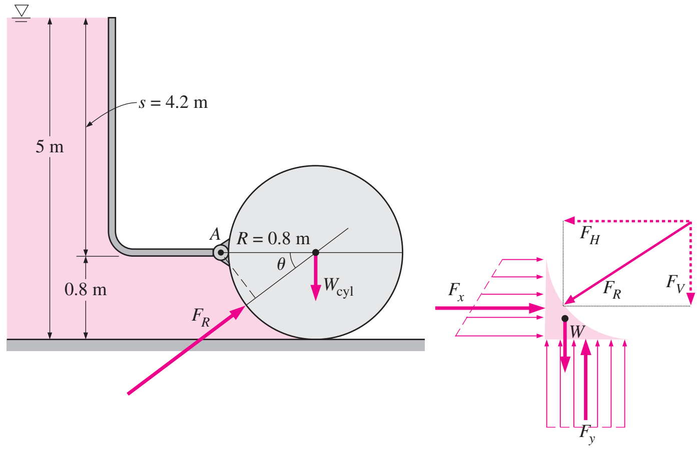


FIGURE 3–36
Schematic for Example 3–9 and the free-body diagram of the fluid underneath the cylinder.

Weight of fluid block per m length (downward):

$$W = mg = \rho g V = \rho g (R^2 - \pi R^2/4)(1\text{ m})$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.8\text{ m})^2(1 - \pi/4)(1\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 1.3\text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9\text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3\text{ kN}}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.4° with the horizontal.

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R \sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R \sin\theta = (52.3\text{ kN})\sin 46.4° = \mathbf{37.9\text{ kN}}$$

Discussion The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of 1921 kg/m^3 for the material of the cylinder.

3–7 ▪ BUOYANCY AND STABILITY

It is a common experience that an object feels lighter and weighs less in a liquid than it does in air. This can be demonstrated easily by weighing a heavy object in water by a waterproof spring scale. Also, objects made of wood or other light materials float on water. These and other observations suggest that a fluid exerts an upward force on a body immersed in it. This force that tends to lift the body is called the **buoyant force** and is denoted by F_B.

The buoyant force is caused by the increase of pressure in a fluid with depth. Consider, for example, a flat plate of thickness h submerged in a liquid of density ρ_f parallel to the free surface, as shown in Fig. 3–37. The area of the top (and also bottom) surface of the plate is A, and its distance to the free surface is s. The pressures at the top and bottom surfaces of the plate are $\rho_f gs$ and $\rho_f g(s + h)$, respectively. Then the hydrostatic force $F_{\text{top}} = \rho_f gsA$ acts downward on the top surface, and the larger force $F_{\text{bottom}} = \rho_f g(s + h)A$ acts upward on the bottom surface of the plate. The difference between these two forces is a net upward force, which is the *buoyant force,*

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s + h)A - \rho_f gsA = \rho_f ghA = \rho_f gV \qquad \textbf{(3–32)}$$

where $V = hA$ is the volume of the plate. But the relation $\rho_f gV$ is simply the weight of the liquid whose volume is equal to the volume of the plate. Thus, we conclude that *the buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.* Note that the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body.

The relation in Eq. 3–32 is developed for a simple geometry, but it is valid for any body regardless of its shape. This can be shown mathematically by a force balance, or simply by this argument: Consider an arbitrarily shaped solid body submerged in a fluid at rest and compare it to a body of fluid of the same shape indicated by dotted lines at the same distance from the free surface (Fig. 3–38). The buoyant forces acting on these two bodies are the same since the pressure distributions, which depend only on depth, are the same at the boundaries of both. The imaginary fluid body is in static equilibrium, and thus the net force and net moment acting on it are zero. Therefore, the upward buoyant force must be equal to the weight of the imaginary fluid body whose volume is equal to the volume of the solid body. Further, the weight and the buoyant force must have the same line of action to have a zero moment. This is known as **Archimedes' principle,** after the Greek mathematician Archimedes (287–212 BC), and is expressed as

> The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body. That is,

$$F_B = W \;\rightarrow\; \rho_f gV_{\text{sub}} = \rho_{\text{ave, body}} gV_{\text{total}} \;\rightarrow\; \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{ave, body}}}{\rho_f} \qquad \textbf{(3–33)}$$

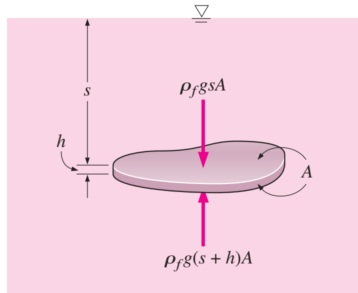


FIGURE 3–37
A flat plate of uniform thickness h submerged in a liquid parallel to the free surface.

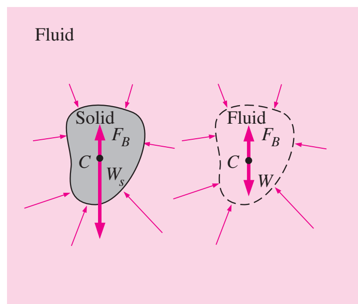


FIGURE 3–38
The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The buoyant force F_B acts upward through the centroid C of the displaced volume and is equal in magnitude to the weight W of the displaced fluid, but is opposite in direction. For a solid of uniform density, its weight W_s also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces. (Here $W_s > W$ and thus $W_s > F_B$; this solid body would sink.)

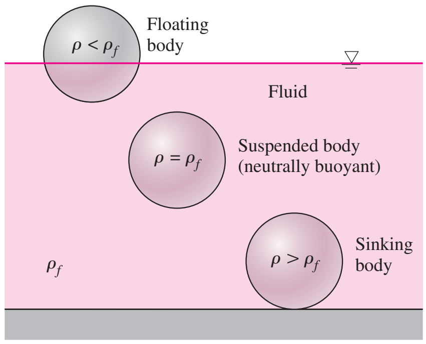


FIGURE 3–39
A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its density relative to the density of the fluid.

Therefore, the submerged volume fraction of a floating body is equal to the ratio of the average density of the body to the density of the fluid. Note that when the density ratio is equal to or greater than one, the floating body becomes completely submerged.

It follows from these discussions that a body immersed in a fluid (1) remains at rest at any point in the fluid when its density is equal to the density of the fluid, (2) sinks to the bottom when its density is greater than the density of the fluid, and (3) rises to the surface of the fluid and floats when the density of the body is less than the density of the fluid (Fig. 3–39).

The buoyant force is proportional to the density of the fluid, and thus we might think that the buoyant force exerted by gases such as air is negligible. This is certainly the case in general, but there are significant exceptions. For example, the volume of a person is about 0.1 m^3, and taking the density of air to be 1.2 kg/m^3, the buoyant force exerted by air on the person is

$$F_B = \rho_f g V = (1.2 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}^3) \cong 1.2 \text{ N}$$

The weight of an 80-kg person is $80 \times 9.81 = 788$ N. Therefore, ignoring the buoyancy in this case results in an error in weight of just 0.15 percent, which is negligible. But the buoyancy effects in gases dominate some important natural phenomena such as the rise of warm air in a cooler environment and thus the onset of natural convection currents, the rise of hot-air or helium balloons, and air movements in the atmosphere. A helium balloon, for example, rises as a result of the buoyancy effect until it reaches an altitude where the density of air (which decreases with altitude) equals the density of helium in the balloon—assuming the balloon does not burst by then, and ignoring the weight of the balloon's skin.

Archimedes' principle is also used in modern geology by considering the continents to be floating on a sea of magma.

EXAMPLE 3–10 **Measuring Specific Gravity by a Hydrometer**

If you have a seawater aquarium, you have probably used a small cylindrical glass tube with some lead-weight at its bottom to measure the salinity of the water by simply watching how deep the tube sinks. Such a device that floats in a vertical position and is used to measure the specific gravity of a liquid is called a *hydrometer* (Fig. 3–40). The top part of the hydrometer extends above the liquid surface, and the divisions on it allow one to read the specific gravity directly. The hydrometer is calibrated such that in pure water it reads exactly 1.0 at the air–water interface. (*a*) Obtain a relation for the specific gravity of a liquid as a function of distance Δz from the mark corresponding to pure water and (*b*) determine the mass of lead that must be poured into a 1-cm-diameter, 20-cm-long hydrometer if it is to float halfway (the 10-cm mark) in pure water.

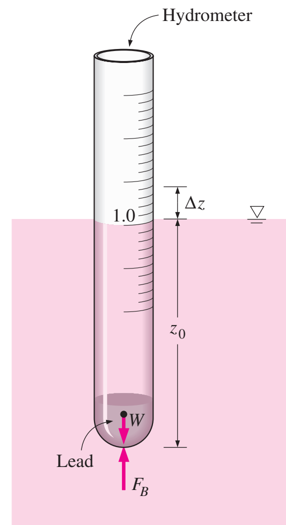


FIGURE 3–40
Schematic for Example 3–10.

SOLUTION The specific gravity of a liquid is to be measured by a hydrometer. A relation between specific gravity and the vertical distance from the reference level is to be obtained, and the amount of lead that needs to be added into the tube for a certain hydrometer is to be determined.

Assumptions **1** The weight of the glass tube is negligible relative to the weight of the lead added. **2** The curvature of the tube bottom is disregarded.

Properties We take the density of pure water to be 1000 kg/m^3.

Analysis (*a*) Noting that the hydrometer is in static equilibrium, the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer. In pure water, let the vertical distance between the bottom of the hydrometer and the free surface of water be z_0. Setting $F_B = W$ in this case gives

$$W_{\text{hydro}} = F_{B,w} = \rho_w g V_{\text{sub}} = \rho_w g A z_0 \qquad \textbf{(1)}$$

where A is the cross-sectional area of the tube, and ρ_w is the density of pure water.

In a fluid lighter than water ($\rho_f < \rho_w$), the hydrometer will sink deeper, and the liquid level will be a distance of Δz above z_0. Again setting $F_B = W$ gives

$$W_{\text{hydro}} = F_{B,f} = \rho_f g V_{\text{sub}} = \rho_f g A(z_0 + \Delta z) \qquad \textbf{(2)}$$

This relation is also valid for fluids heavier than water by taking the Δz below z_0 to be a negative quantity. Setting Eqs. (1) and (2) here equal to each other since the weight of the hydrometer is constant and rearranging gives

$$\rho_w g A z_0 = \rho_f g A(z_0 + \Delta z) \quad \rightarrow \quad \text{SG}_f = \frac{\rho_f}{\rho_w} = \frac{z_0}{z_0 + \Delta z}$$

which is the relation between the specific gravity of the fluid and Δz. Note that z_0 is constant for a given hydrometer and Δz is negative for fluids heavier than pure water.

(*b*) Disregarding the weight of the glass tube, the amount of lead that needs to be added to the tube is determined from the requirement that the weight of the lead be equal to the buoyant force. When the hydrometer is floating with half of it submerged in water, the buoyant force acting on it is

$$F_B = \rho_w g V_{\text{sub}}$$

Equating F_B to the weight of lead gives

$$W = mg = \rho_w g V_{\text{sub}}$$

Solving for m and substituting, the mass of lead is determined to be

$$m = \rho_w V_{\text{sub}} = \rho_w(\pi R^2 h_{\text{sub}}) = (1000\text{ kg/m}^3)[\pi(0.005\text{ m})^2(0.1\text{ m})] = \mathbf{0.00785\text{ kg}}$$

Discussion Note that if the hydrometer were required to sink only 5 cm in water, the required mass of lead would be one-half of this amount. Also, the assumption that the weight of the glass tube is negligible needs to be checked since the mass of lead is only 7.85 g.

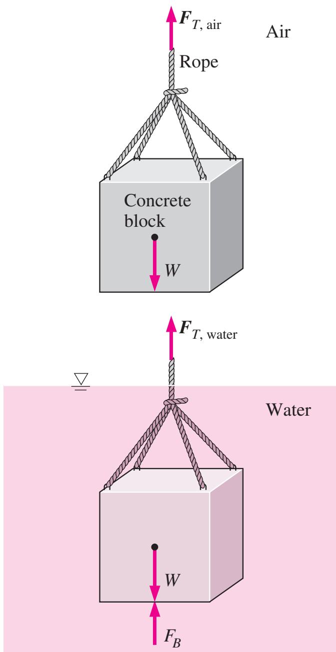


FIGURE 3–41
Schematic for Example 3–11.

EXAMPLE 3–11 Weight Loss of an Object in Seawater

A crane is used to lower weights into the sea (density = 1025 kg/m^3) for an underwater construction project (Fig. 3–41). Determine the tension in the rope of the crane due to a rectangular 0.4-m × 0.4-m × 3-m concrete block (density = 2300 kg/m^3) when it is (*a*) suspended in the air and (*b*) completely immersed in water.

SOLUTION A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is in water.

Assumptions **1** The buoyancy of air is negligible. **2** The weight of the ropes is negligible.

Properties The densities are given to be 1025 kg/m^3 for seawater and 2300 kg/m^3 for concrete.

Analysis (*a*) Consider the free-body diagram of the concrete block. The forces acting on the concrete block in air are its weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$V = (0.4\text{ m})(0.4\text{ m})(3\text{ m}) = 0.48\text{ m}^3$$

$$F_{T,\text{ air}} = W = \rho_{\text{concrete}}\, gV$$

$$= (2300\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.48\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{10.8\text{ kN}}$$

(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upward. The force balance in this case gives

$$F_B = \rho_f\, gV = (1025\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.48\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{4.8\text{ kN}}$$

$$F_{T,\text{ water}} = W - F_B = 10.8 - 4.8 = \mathbf{6.0\text{ kN}}$$

Discussion Note that the weight of the concrete block, and thus the tension of the rope, decreases by (10.8 − 6.0)/10.8 = 55 percent in water.

Stability of Immersed and Floating Bodies

An important application of the buoyancy concept is the assessment of the stability of immersed and floating bodies with no external attachments. This topic is of great importance in the design of ships and submarines (Fig. 3–42). Here we provide some general qualitative discussions on vertical and rotational stability.

We use the "ball on the floor" analogy to explain the fundamental concepts of stability and instability. Shown in Fig. 3–43 are three balls at rest on the floor. Case (*a*) is **stable** since any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position. Case (*b*) is **neutrally stable** because if someone moves the ball to the right or left, it would stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away. Case (*c*) is a situation in which the ball may be at rest at the moment, but any disturbance, even an infinitesimal one, causes the ball to roll off the hill—it does not return to its original position; rather it *diverges* from it. This situation is **unstable.** What about a case where the ball is on an *inclined* floor? It is not really appropriate to discuss stability for this case since the ball is not in a state of equilibrium. In other words, it cannot be at rest and would roll down the hill even without any disturbance.

For an immersed or floating body in static equilibrium, the weight and the buoyant force acting on the body balance each other, and such bodies are inherently stable in the *vertical direction.* If an immersed neutrally buoyant body is raised or lowered to a different depth, the body will remain in equilibrium at that location. If a floating body is raised or lowered somewhat by a vertical force, the body will return to its original position as soon as the external effect is removed. Therefore, a floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.

The *rotational stability* of an *immersed body* depends on the relative locations of the *center of gravity G* of the body and the *center of buoyancy B,* which is the centroid of the displaced volume. An immersed body is *stable* if the body is bottom-heavy and thus point *G* is directly below point *B* (Fig. 3–44). A rotational disturbance of the body in such cases produces a *restoring moment* to return the body to its original stable position. Thus, a stable design for a submarine calls for the engines and the cabins for the crew to be located at the lower half in order to shift the weight to the bottom as much as possible. Hot-air or helium balloons (which can be viewed as being immersed in air) are also stable since the cage that carries the load is at the bottom. An immersed body whose center of gravity *G* is directly above point *B* is *unstable,* and any disturbance will cause this body to turn upside down. A body for which *G* and *B* coincide is *neutrally stable.* This is the case for bodies whose density is constant throughout. For such bodies, there is no tendency to overturn or right themselves.

What about a case where the center of gravity is not vertically aligned with the center of buoyancy (Fig. 3–45)? It is not really appropriate to discuss stability for this case since the body is not in a state of equilibrium. In other words, it cannot be at rest and would rotate toward its stable state even without any disturbance. The restoring moment in the case shown in Fig. 3–45 is counterclockwise and causes the body to rotate counterclockwise so as to align point *G* vertically with point *B*. Note that there may be some oscillation, but eventually the body settles down at its stable equilibrium state [case (*a*) of Fig. 3–44]. The stability of the body of Fig. 3–45 is analogous to that of the ball on an inclined floor. Can you predict what would happen if the weight in the body of Fig. 3–45 were on the opposite side of the body?

The rotational stability criteria are similar for *floating bodies.* Again, if the floating body is bottom-heavy and thus the center of gravity *G* is directly below the center of buoyancy *B,* the body is always stable. But unlike immersed bodies, a floating body may still be stable when *G* is directly above *B* (Fig. 3–46). This is because the centroid of the displaced volume shifts to the side to a point B' during a rotational disturbance while the center of gravity *G* of the body remains unchanged. If point B' is sufficiently far,

FIGURE 3–42
For floating bodies such as ships, stability is an important consideration for safety.
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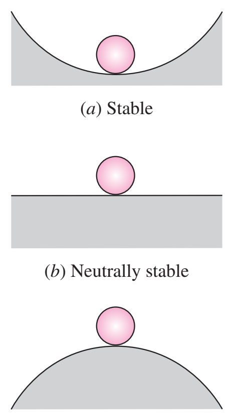


(*a*) Stable

(*b*) Neutrally stable

(*c*) Unstable

FIGURE 3–43
Stability is easily understood by analyzing a ball on the floor.

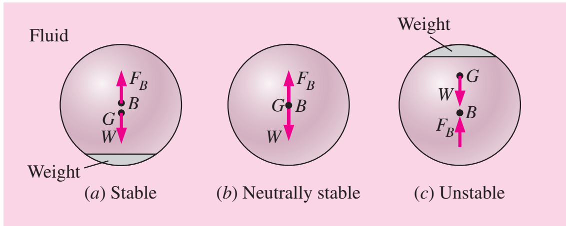


FIGURE 3–44
An immersed neutrally buoyant body is (*a*) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body, (*b*) neutrally stable if *G* and *B* are coincident, and (*c*) unstable if *G* is directly above *B*.

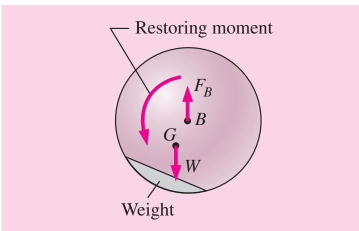


FIGURE 3–45
When the center of gravity *G* of an immersed neutrally buoyant body is not vertically aligned with the center of buoyancy *B* of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance.

these two forces create a restoring moment and return the body to the original position. A measure of stability for floating bodies is the **metacentric height** *GM,* which is the distance between the center of gravity *G* and the metacenter *M*—the intersection point of the lines of action of the buoyant force through the body before and after rotation. The metacenter may be considered to be a fixed point for most hull shapes for small rolling angles up to about 20°. A floating body is stable if point *M* is above point *G,* and thus *GM* is positive, and unstable if point *M* is below point *G,* and thus *GM* is negative. In the latter case, the weight and the buoyant force acting on the tilted body generate an overturning moment instead of a restoring moment, causing the body to capsize. The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

As already discussed, a boat can tilt to some maximum angle without capsizing, but beyond that angle it overturns (and sinks). We make a final analogy between the stability of floating objects and the stability of a ball rolling along the floor. Namely, imagine the ball in a trough between two hills (Fig. 3–47). The ball returns to its stable equilibrium position after being perturbed—up to a limit. If the perturbation amplitude is too great, the ball rolls down the opposite side of the hill and does not return to its equilibrium position. This situation is described as stable up to some limiting level of disturbance, but unstable beyond.

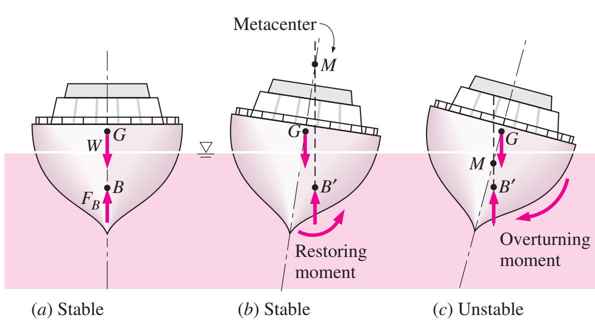


FIGURE 3–46
A floating body is *stable* if the body is bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or if the metacenter *M* is above point *G*. However, the body is *unstable* if point *M* is below point *G*.

3–8 ■ FLUIDS IN RIGID-BODY MOTION

We showed in Section 3–1 that pressure at a given point has the same magnitude in all directions, and thus it is a *scalar* function. In this section we obtain relations for the variation of pressure in fluids moving like a solid body with or without acceleration in the absence of any shear stresses (i.e., no motion between fluid layers relative to each other).

Many fluids such as milk and gasoline are transported in tankers. In an accelerating tanker, the fluid rushes to the back, and some initial splashing occurs. But then a new free surface (usually nonhorizontal) is formed, each fluid particle assumes the same acceleration, and the entire fluid moves like a rigid body. No shear stresses develop within the fluid body since there is no deformation and thus no change in shape. Rigid-body motion of a fluid also occurs when the fluid is contained in a tank that rotates about an axis.

Consider a differential rectangular fluid element of side lengths dx, dy, and dz in the x-, y-, and z-directions, respectively, with the z-axis being upward in the vertical direction (Fig. 3–48). Noting that the differential fluid element behaves like a *rigid body, Newton's second law of motion* for this element can be expressed as

$$\delta\vec{F} = \delta m \cdot \vec{a} \tag{3–34}$$

where $\delta m = \rho\, dV = \rho\, dx\, dy\, dz$ is the mass of the fluid element, $\vec{a}$ is the acceleration, and $\delta\vec{F}$ is the net force acting on the element.

The forces acting on the fluid element consist of *body forces* such as gravity that act throughout the entire body of the element and are proportional to the volume of the body (and also electrical and magnetic forces, which will not be considered in this text), and *surface forces* such as the pressure forces that act on the surface of the element and are proportional to the surface area (shear stresses are also surface forces, but they do not apply in this case since the relative positions of fluid elements remain unchanged). The surface forces appear as the fluid element is isolated from its surroundings for analysis, and the effect of the detached body is replaced by a force at that location. Note that pressure represents the compressive force applied on the fluid element by the surrounding fluid and is always directed to the surface.

Taking the pressure at the center of the element to be P, the pressures at the top and bottom surfaces of the element can be expressed as $P + (\partial P/\partial z)\, dz/2$ and $P - (\partial P/\partial z)\, dz/2$, respectively. Noting that the pressure force acting on a surface is equal to the average pressure multiplied by the surface area, the net surface force acting on the element in the z-direction is the difference between the pressure forces acting on the bottom and top faces,

$$\delta F_{S,z} = \left(P - \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy - \left(P + \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy = -\frac{\partial P}{\partial z}\, dx\, dy\, dz \tag{3–35}$$

Similarly, the net surface forces in the x- and y-directions are

$$\delta F_{S,x} = -\frac{\partial P}{\partial x}\, dx\, dy\, dz \quad \text{and} \quad \delta F_{S,y} = -\frac{\partial P}{\partial y}\, dx\, dy\, dz \tag{3–36}$$

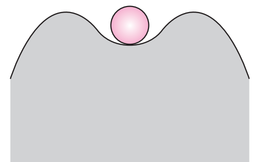


FIGURE 3–47
A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.

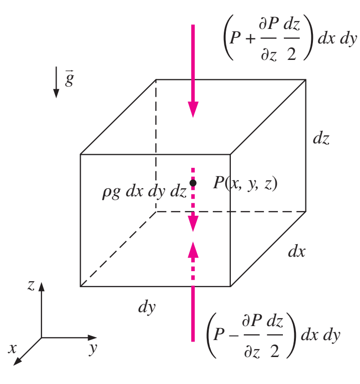


FIGURE 3–48
The surface and body forces acting on a differential fluid element in the vertical direction.

Then the surface force (which is simply the pressure force) acting on the entire element can be expressed in vector form as

$$\delta \vec{F}_S = \delta F_{S,x}\vec{i} + \delta F_{S,y}\vec{j} + \delta F_{S,z}\vec{k}$$
$$= -\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right) dx\,dy\,dz = -\vec{\nabla}P\,dx\,dy\,dz \tag{3–37}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the unit vectors in the x-, y-, and z-directions, respectively, and

$$\vec{\nabla}P = \frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k} \tag{3–38}$$

is the *pressure gradient*. Note that the $\vec{\nabla}$ or "del" is a vector operator that is used to express the gradients of a scalar function compactly in vector form. Also, the *gradient* of a scalar function is expressed in a given *direction* and thus it is a *vector* quantity.

The only body force acting on the fluid element is the weight of the element acting in the negative z-direction, and it is expressed as $\delta F_{B,z} = -g\delta m = -\rho g\,dx\,dy\,dz$ or in vector form as

$$\delta \vec{F}_{B,z} = -g\delta m\vec{k} = -\rho g\,dx\,dy\,dz\vec{k} \tag{3–39}$$

Then the total force acting on the element becomes

$$\delta \vec{F} = \delta \vec{F}_S + \delta \vec{F}_B = -(\vec{\nabla}P + \rho g\vec{k})\,dx\,dy\,dz \tag{3–40}$$

Substituting into Newton's second law of motion $\delta \vec{F} = \delta m \cdot \vec{a} = \rho\,dx\,dy\,dz \cdot \vec{a}$ and canceling $dx\,dy\,dz$, the general **equation of motion** for a fluid that acts as a rigid body (no shear stresses) is determined to be

Rigid-body motion of fluids:
$$\vec{\nabla}P + \rho g\vec{k} = -\rho\vec{a} \tag{3–41}$$

Resolving the vectors into their components, this relation can be expressed more explicitly as

$$\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k} + \rho g\vec{k} = -\rho(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}) \tag{3–42}$$

or, in scalar form in the three orthogonal directions, as

Accelerating fluids:
$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = -\rho a_y, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z) \tag{3–43}$$

where a_x, a_y, and a_z are accelerations in the x-, y-, and z-directions, respectively.

Special Case 1: Fluids at Rest

For fluids at rest or moving on a straight path at constant velocity, all components of acceleration are zero, and the relations in Eqs. 3–43 reduce to

Fluids at rest:
$$\frac{\partial P}{\partial x} = 0, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{dP}{dz} = -\rho g \tag{3–44}$$

which confirm that, in fluids at rest, the pressure remains constant in any horizontal direction (P is independent of x and y) and varies only in the vertical direction as a result of gravity [and thus $P = P(z)$]. These relations are applicable for both compressible and incompressible fluids.

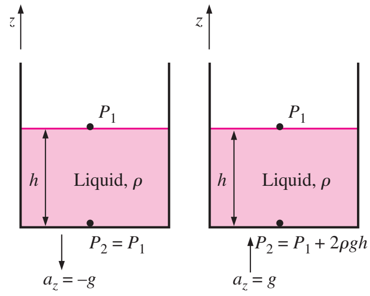
Special Case 2: Free Fall of a Fluid Body

A freely falling body accelerates under the influence of gravity. When the air resistance is negligible, the acceleration of the body equals the gravitational acceleration, and acceleration in any horizontal direction is zero. Therefore, $a_x = a_y = 0$ and $a_z = -g$. Then the equations of motion for accelerating fluids (Eqs. 3–43) reduce to

Free-falling fluids: $$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0 \quad \rightarrow \quad P = \text{constant} \tag{3–45}$$

Therefore, in a frame of reference moving with the fluid, it behaves like it is in an environment with zero gravity. Also, the gage pressure in a drop of liquid in free fall is zero throughout. (Actually, the gage pressure is slightly above zero due to surface tension, which holds the drop intact.)

When the direction of motion is reversed and the fluid is forced to accelerate vertically with $a_z = +g$ by placing the fluid container in an elevator or a space vehicle propelled upward by a rocket engine, the pressure gradient in the z-direction is $\partial P/\partial z = -2\rho g$. Therefore, the pressure difference across a fluid layer now doubles relative to the stationary fluid case (Fig. 3–49).



(*a*) Free fall of a liquid

(*b*) Upward acceleration of a liquid with $a_z = +g$

FIGURE 3–49
The effect of acceleration on the pressure of a liquid during free fall and upward acceleration.

Acceleration on a Straight Path

Consider a container partially filled with a liquid. The container is moving on a straight path with a constant acceleration. We take the projection of the path of motion on the horizontal plane to be the x-axis, and the projection on the vertical plane to be the z-axis, as shown in Fig. 3–50. The x- and z-components of acceleration are a_x and a_z. There is no movement in the y-direction, and thus the acceleration in that direction is zero, $a_y = 0$. Then the equations of motion for accelerating fluids (Eqs. 3–43) reduce to

$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z) \tag{3–46}$$

Therefore, pressure is independent of y. Then the total differential of $P = P(x, z)$, which is $(\partial P/\partial x)\, dx + (\partial P/\partial z)\, dz$, becomes

$$dP = -\rho a_x\, dx - \rho(g + a_z)\, dz \tag{3–47}$$

For ρ = constant, the pressure difference between two points 1 and 2 in the fluid is determined by integration to be

$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \tag{3–48}$$

Taking point 1 to be the origin ($x = 0$, $z = 0$) where the pressure is P_0 and point 2 to be any point in the fluid (no subscript), the pressure distribution can be expressed as

Pressure variation: $$P = P_0 - \rho a_x x - \rho(g + a_z)z \tag{3–49}$$

The vertical rise (or drop) of the free surface at point 2 relative to point 1 can be determined by choosing both 1 and 2 on the free surface (so that $P_1 = P_2$), and solving Eq. 3–48 for $z_2 - z_1$ (Fig. 3–51),

Vertical rise of surface: $$\Delta z_s = z_{s2} - z_{s1} = -\frac{a_x}{g + a_z}(x_2 - x_1) \tag{3–50}$$

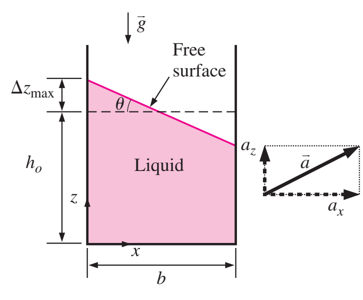


FIGURE 3–50
Rigid-body motion of a liquid in a linearly accelerating tank.

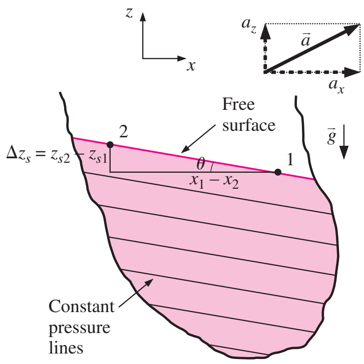


FIGURE 3–51
Lines of constant pressure (which are the projections of the surfaces of constant pressure on the *xz*-plane) in a linearly accelerating liquid, and the vertical rise.

where z_s is the *z*-coordinate of the liquid's free surface. The equation for surfaces of constant pressure, called **isobars,** is obtained from Eq. 3–47 by setting $dP = 0$ and replacing z by z_{isobar}, which is the *z*-coordinate (the vertical distance) of the surface as a function of *x*. It gives

Surfaces of constant pressure:
$$\frac{dz_{isobar}}{dx} = -\frac{a_x}{g + a_z} = \text{constant} \qquad \textbf{(3–51)}$$

Thus we conclude that the isobars (including the free surface) in an incompressible fluid with constant acceleration in linear motion are parallel surfaces whose slope in the *xz*-plane is

Slope of isobars:
$$\text{Slope} = \frac{dz_{isobar}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta \qquad \textbf{(3–52)}$$

Obviously, the free surface of such a fluid is a *plane* surface, and it is inclined unless $a_x = 0$ (the acceleration is in the vertical direction only). Also, the conservation of mass together with the assumption of incompressibility (ρ = *constant*) requires that the volume of the fluid remain constant before and during acceleration. Therefore, the rise of fluid level on one side must be balanced by a drop of fluid level on the other side.

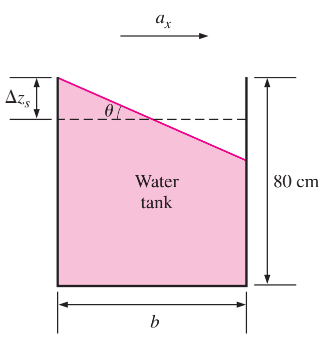


FIGURE 3–52
Schematic for Example 3–12.

EXAMPLE 3–12 **Overflow from a Water Tank During Acceleration**

An 80-cm-high fish tank of cross section 2 m × 0.6 m that is initially filled with water is to be transported on the back of a truck (Fig. 3–52). The truck accelerates from 0 to 90 km/h in 10 s. If it is desired that no water spills during acceleration, determine the allowable initial water height in the tank. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion?

SOLUTION A fish tank is to be transported on a truck. The allowable water height to avoid spill of water during acceleration and the proper orientation are to be determined.

Assumptions **1** The road is horizontal during acceleration so that acceleration has no vertical component ($a_z = 0$). **2** Effects of splashing, braking, driving over bumps, and climbing hills are assumed to be secondary and are not considered. **3** The acceleration remains constant.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction, and the origin to be the lower left corner of the tank. Noting that the truck goes from 0 to 90 km/h in 10 s, the acceleration of the truck is

$$a_x = \frac{\Delta V}{\Delta t} = \frac{(90 - 0)\ \text{km/h}}{10\ \text{s}}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 2.5\ \text{m/s}^2$$

The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{2.5}{9.81 + 0} = 0.255 \qquad (\text{and thus } \theta = 14.3°)$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midplane experiences no rise or drop during acceleration since it is a plane of symmetry. Then the vertical rise at the back of the tank relative to the midplane for the two possible orientations becomes

Case 1: The long side is parallel to the direction of motion:

$$\Delta z_{s1} = (b_1/2)\tan\theta = [(2\text{ m})/2] \times 0.255 = 0.255\text{ m} = \mathbf{25.5\text{ cm}}$$

Case 2: The short side is parallel to the direction of motion:

$$\Delta z_{s2} = (b_2/2)\tan\theta = [(0.6\text{ m})/2] \times 0.255 = 0.076\text{ m} = \mathbf{7.6\text{ cm}}$$

Therefore, assuming tipping is not a problem, the tank should definitely be oriented such that its short side is parallel to the direction of motion. Emptying the tank such that its free surface level drops just 7.6 cm in this case will be adequate to avoid spilling during acceleration.

Discussion Note that the orientation of the tank is important in controlling the vertical rise. Also, the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

Rotation in a Cylindrical Container

We know from experience that when a glass filled with water is rotated about its axis, the fluid is forced outward as a result of the so-called centrifugal force, and the free surface of the liquid becomes concave. This is known as the *forced vortex motion*.

Consider a vertical cylindrical container partially filled with a liquid. The container is now rotated about its axis at a constant angular velocity of ω, as shown in Fig. 3–53. After initial transients, the liquid will move as a rigid body together with the container. There is no deformation, and thus there can be no shear stress, and every fluid particle in the container moves with the same angular velocity.

This problem is best analyzed in cylindrical coordinates (r, θ, z), with z taken along the centerline of the container directed from the bottom toward the free surface, since the shape of the container is a cylinder, and the fluid particles undergo a circular motion. The centripetal acceleration of a fluid particle rotating with a constant angular velocity of ω at a distance r from the axis of rotation is $r\omega^2$ and is directed radially toward the axis of rotation (negative r-direction). That is, $a_r = -r\omega^2$. There is symmetry about the z-axis, which is the axis of rotation, and thus there is no θ dependence. Then $P = P(r, z)$ and $a_\theta = 0$. Also, $a_z = 0$ since there is no motion in the z-direction.

Then the equations of motion for rotating fluids (Eqs. 3–43) reduce to

$$\frac{\partial P}{\partial r} = \rho r\omega^2, \qquad \frac{\partial P}{\partial \theta} = 0, \qquad \text{and} \qquad \frac{\partial P}{\partial z} = -\rho g \tag{3–53}$$

Then the total differential of $P = P(r, z)$, which is $dP = (\partial P/\partial r)dr + (\partial P/\partial z)dz$, becomes

$$dP = \rho r\omega^2\, dr - \rho g\, dz \tag{3–54}$$

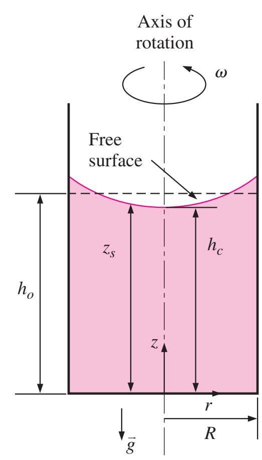


FIGURE 3–53
Rigid-body motion of a liquid in a rotating vertical cylindrical container.

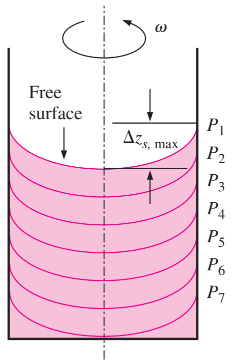


FIGURE 3–54
Surfaces of constant pressure in a rotating liquid.

The equation for surfaces of constant pressure is obtained by setting $dP = 0$ and replacing z by z_{isobar}, which is the z-value (the vertical distance) of the surface as a function of r. It gives

$$\frac{dz_{\text{isobar}}}{dr} = \frac{r\omega^2}{g} \tag{3–55}$$

Integrating, the equation for the surfaces of constant pressure is determined to be

Surfaces of constant pressure: $$z_{\text{isobar}} = \frac{\omega^2}{2g} r^2 + C_1 \tag{3–56}$$

which is the equation of a *parabola*. Thus we conclude that the surfaces of constant pressure, including the free surface, are *paraboloids of revolution* (Fig. 3–54).

The value of the integration constant C_1 is different for different paraboloids of constant pressure (i.e., for different isobars). For the free surface, setting $r = 0$ in Eq. 3–56 gives $z_{\text{isobar}}(0) = C_1 = h_c$, where h_c is the distance of the free surface from the bottom of the container along the axis of rotation (Fig. 3–53). Then the equation for the free surface becomes

$$z_s = \frac{\omega^2}{2g} r^2 + h_c \tag{3–57}$$

where z_s is the distance of the free surface from the bottom of the container at radius r. The underlying assumption in this analysis is that there is sufficient liquid in the container so that the entire bottom surface remains covered with liquid.

The volume of a cylindrical shell element of radius r, height z_s, and thickness dr is $dV = 2\pi r z_s\, dr$. Then the volume of the paraboloid formed by the free surface is

$$V = \int_{r=0}^{R} 2\pi z_s r\, dr = 2\pi \int_{r=0}^{R} \left(\frac{\omega^2}{2g} r^2 + h_c\right) r\, dr = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c\right) \tag{3–58}$$

Since mass is conserved and density is constant, this volume must be equal to the original volume of the fluid in the container, which is

$$V = \pi R^2 h_0 \tag{3–59}$$

where h_0 is the original height of the fluid in the container with no rotation. Setting these two volumes equal to each other, the height of the fluid along the centerline of the cylindrical container becomes

$$h_c = h_0 - \frac{\omega^2 R^2}{4g} \tag{3–60}$$

Then the equation of the free surface becomes

Free surface: $$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2) \tag{3–61}$$

The maximum vertical height occurs at the edge where $r = R$, and the *maximum height difference* between the edge and the center of the free surface is determined by evaluating z_s at $r = R$ and also at $r = 0$, and taking their difference,

Maximum height difference:
$$\Delta z_{s,\max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g}R^2 \quad \textbf{(3–62)}$$

When ρ = constant, the pressure difference between two points 1 and 2 in the fluid is determined by integrating $dP = \rho r\omega^2\,dr - \rho g\,dz$. This yields

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1) \quad \textbf{(3–63)}$$

Taking point 1 to be the origin ($r = 0$, $z = 0$) where the pressure is P_0 and point 2 to be any point in the fluid (no subscript), the pressure distribution can be expressed as

Pressure variation:
$$P = P_0 + \frac{\rho\omega^2}{2}r^2 - \rho g z \quad \textbf{(3–64)}$$

Note that at a fixed radius, the pressure varies hydrostatically in the vertical direction, as in a fluid at rest. For a fixed vertical distance z, the pressure varies with the square of the radial distance r, increasing from the centerline toward the outer edge. In any horizontal plane, the pressure difference between the center and edge of the container of radius R is $\Delta P = \rho\omega^2R^2/2$.

EXAMPLE 3–13 Rising of a Liquid During Rotation

A 20-cm-diameter, 60-cm-high vertical cylindrical container, shown in Fig. 3–55, is partially filled with 50-cm-high liquid whose density is 850 kg/m^3. Now the cylinder is rotated at a constant speed. Determine the rotational speed at which the liquid will start spilling from the edges of the container.

SOLUTION A vertical cylindrical container partially filled with a liquid is rotated. The angular speed at which the liquid will start spilling is to be determined.

Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** The bottom surface of the container remains covered with liquid during rotation (no dry spots).

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin ($r = 0$, $z = 0$), the equation for the free surface of the liquid is given as

$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

Then the vertical height of the liquid at the edge of the container where $r = R$ becomes

$$z_s(R) = h_0 + \frac{\omega^2R^2}{4g}$$

where $h_0 = 0.5$ m is the original height of the liquid before rotation. Just before the liquid starts spilling, the height of the liquid at the edge of the container equals the height of the container, and thus $z_s(R) = 0.6$ m. Solving the

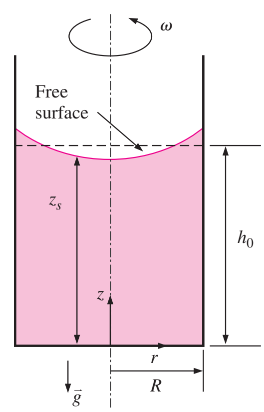


FIGURE 3–55
Schematic for Example 3–13.

last equation for ω and substituting, the maximum rotational speed of the container is determined to be

$$\omega = \sqrt{\frac{4g[z_s(R) - h_0]}{R^2}} = \sqrt{\frac{4(9.81\ \text{m/s}^2)[(0.6 - 0.5)\ \text{m}]}{(0.1\ \text{m})^2}} = \mathbf{19.8\ rad/s}$$

Noting that one complete revolution corresponds to 2π rad, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

$$\dot{n} = \frac{\omega}{2\pi} = \frac{19.8\ \text{rad/s}}{2\pi\ \text{rad/rev}}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{189\ rpm}$$

Therefore, the rotational speed of this container should be limited to 189 rpm to avoid any spill of liquid as a result of the centrifugal effect.

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. We should also verify that our assumption of no dry spots is valid. The liquid height at the center is

$$z_s(0) = h_0 - \frac{\omega^2 R^2}{4g} = 0.4\ \text{m}$$

Since $z_s(0)$ is positive, our assumption is validated.

SUMMARY

The normal force exerted by a fluid per unit area is called *pressure,* and its unit is the *pascal,* 1 Pa ≡ 1 N/m^2. The pressure relative to absolute vacuum is called the *absolute pressure,* and the difference between the absolute pressure and the local atmospheric pressure is called the *gage pressure.* Pressures below atmospheric pressure are called *vacuum pressures.* The absolute, gage, and vacuum pressures are related by

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}}$$

The pressure at a point in a fluid has the same magnitude in all directions. The variation of pressure with elevation in a fluid at rest is given by

$$\frac{dP}{dz} = -\rho g$$

where the positive z-direction is taken to be upward. When the density of the fluid is constant, the pressure difference across a fluid layer of thickness Δz is

$$\Delta P = P_2 - P_1 = \rho g\, \Delta z$$

The absolute and gage pressures in a static liquid open to the atmosphere at a depth h from the free surface are

$$P = P_{\text{atm}} + \rho g h \qquad \text{and} \qquad P_{\text{gage}} = \rho g h$$

The pressure in a fluid at rest remains constant in the horizontal direction. *Pascal's law* states that the pressure applied to a confined fluid increases the pressure throughout by the same amount. The atmospheric pressure is measured by a *barometer* and is given by

$$P_{\text{atm}} = \rho g h$$

where h is the height of the liquid column.

Fluid statics deals with problems associated with fluids at rest, and it is called *hydrostatics* when the fluid is a liquid. The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface and is expressed as

$$F_R = (P_0 + \rho g h_C)A = P_C A = P_{\text{ave}} A$$

where $h_C = y_C \sin\theta$ is the *vertical distance* of the centroid from the free surface of the liquid. The pressure P_0 is usually the atmospheric pressure, which cancels out in most cases since it acts on both sides of the plate. The point of intersection of the line of action of the resultant force and the surface is the *center of pressure.* The vertical location of the line of action of the resultant force is given by

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area.

A fluid exerts an upward force on a body immersed in it. This force is called the *buoyant force* and is expressed as

$$F_B = \rho_f g V$$

where V is the volume of the body. This is known as *Archimedes' principle* and is expressed as: the buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body; it acts upward through the centroid of the displaced volume. With constant density, the buoyant force is independent of the distance of the body from the free surface. For *floating* bodies, the submerged volume fraction of the body is equal to the ratio of the average density of the body to the density of the fluid.

The general *equation of motion* for a fluid that acts as a rigid body is

$$\vec{\nabla} P + \rho g \vec{k} = -\rho \vec{a}$$

When gravity is aligned in the $-z$-direction, it is expressed in scalar form as

$$\frac{\partial P}{\partial x} = -\rho a_x, \qquad \frac{\partial P}{\partial y} = -\rho a_y, \qquad \text{and} \qquad \frac{\partial P}{\partial z} = -\rho(g + a_z)$$

where a_x, a_y, and a_z are accelerations in the x-, y-, and z-directions, respectively. During *linearly accelerating motion* in the xz-plane, the pressure distribution is expressed as

$$P = P_0 - \rho a_x x - \rho(g + a_z)z$$

The surfaces of constant pressure (including the free surface) in a liquid with constant acceleration in linear motion are parallel surfaces whose slope in a xz-plane is

$$\text{Slope} = \frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$

During rigid-body motion of a liquid in a *rotating cylinder*, the surfaces of constant pressure are *paraboloids of revolution*. The equation for the free surface is

$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where z_s is the distance of the free surface from the bottom of the container at radius r and h_0 is the original height of the fluid in the container with no rotation. The variation of pressure in the liquid is expressed as

$$P = P_0 + \frac{\rho\omega^2}{2} r^2 - \rho g z$$

where P_0 is the pressure at the origin ($r = 0$, $z = 0$).

Pressure is a fundamental property, and it is hard to imagine a significant fluid flow problem that does not involve pressure. Therefore, you will see this property in all chapters in the rest of this book. The consideration of hydrostatic forces acting on plane or curved surfaces, however, is mostly limited to this chapter.

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2. C. T. Crowe, J. A. Roberson, and D. F. Elger. *Engineering Fluid Mechanics,* 7th ed. New York: Wiley, 2001.
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PROBLEMS*

Pressure, Manometer, and Barometer

3–1C What is the difference between gage pressure and absolute pressure?

3–2C Explain why some people experience nose bleeding and some others experience shortness of breath at high elevations.

3–3C Someone claims that the absolute pressure in a liquid of constant density doubles when the depth is doubled. Do you agree? Explain.

* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD. Problems with the icon are comprehensive in nature and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

3–4C A tiny steel cube is suspended in water by a string. If the lengths of the sides of the cube are very small, how would you compare the magnitudes of the pressures on the top, bottom, and side surfaces of the cube?

3–5C Express Pascal's law, and give a real-world example of it.

3–6C Consider two identical fans, one at sea level and the other on top of a high mountain, running at identical speeds. How would you compare (*a*) the volume flow rates and (*b*) the mass flow rates of these two fans?

3–7 A vacuum gage connected to a chamber reads 24 kPa at a location where the atmospheric pressure is 92 kPa. Determine the absolute pressure in the chamber.

3–8E A manometer is used to measure the air pressure in a tank. The fluid used has a specific gravity of 1.25, and the differential height between the two arms of the manometer is 28 in. If the local atmospheric pressure is 12.7 psia, determine the absolute pressure in the tank for the cases of the manometer arm with the (*a*) higher and (*b*) lower fluid level being attached to the tank.

3–9 The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. P3–9. Determine the gage pressure of air in the tank if $h_1 = 0.2$ m, $h_2 = 0.3$ m, and $h_3 = 0.46$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

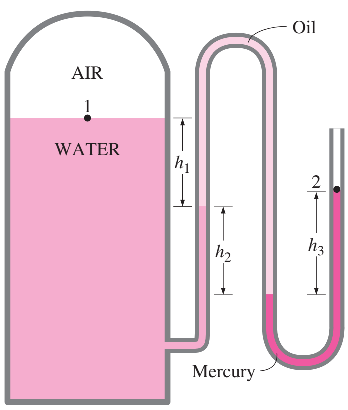

FIGURE P3–9

3–10 Determine the atmospheric pressure at a location where the barometric reading is 750 mmHg. Take the density of mercury to be 13,600 kg/m^3.

3–11 The gage pressure in a liquid at a depth of 3 m is read to be 28 kPa. Determine the gage pressure in the same liquid at a depth of 12 m.

3–12 The absolute pressure in water at a depth of 5 m is read to be 145 kPa. Determine (*a*) the local atmospheric pressure, and (*b*) the absolute pressure at a depth of 5 m in a liquid whose specific gravity is 0.85 at the same location.

3–13E Show that 1 kgf/cm^2 = 14.223 psi.

3–14E A 200-lb man has a total foot imprint area of 72 in^2. Determine the pressure this man exerts on the ground if (*a*) he stands on both feet and (*b*) he stands on one foot.

3–15 Consider a 70-kg woman who has a total foot imprint area of 400 cm^2. She wishes to walk on the snow, but the snow cannot withstand pressures greater than 0.5 kPa. Determine the minimum size of the snowshoes needed (imprint area per shoe) to enable her to walk on the snow without sinking.

3–16 A vacuum gage connected to a tank reads 30 kPa at a location where the barometric reading is 755 mmHg. Determine the absolute pressure in the tank. Take ρ_{Hg} = 13,590 kg/m^3. *Answer:* 70.6 kPa

3–17E A pressure gage connected to a tank reads 50 psi at a location where the barometric reading is 29.1 inHg. Determine the absolute pressure in the tank. Take ρ_{Hg} = 848.4 lbm/ft^3. *Answer:* 64.29 psia

3–18 A pressure gage connected to a tank reads 500 kPa at a location where the atmospheric pressure is 94 kPa. Determine the absolute pressure in the tank.

3–19 The barometer of a mountain hiker reads 930 mbars at the beginning of a hiking trip and 780 mbars at the end. Neglecting the effect of altitude on local gravitational acceleration, determine the vertical distance climbed. Assume an average air density of 1.20 kg/m^3. *Answer:* 1274 m

3–20 The basic barometer can be used to measure the height of a building. If the barometric readings at the top and at the bottom of a building are 730 and 755 mmHg, respectively, determine the height of the building. Assume an average air density of 1.18 kg/m^3.

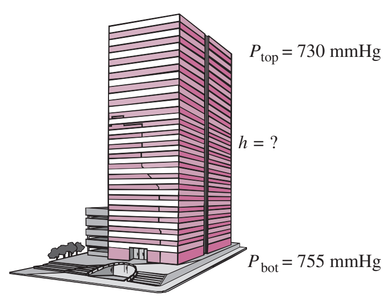

FIGURE P3–20

3–21 EES Solve Prob. 3–20 using EES (or other) software. Print out the entire solution, including the numerical results with proper units, and take the density of mercury to be 13,600 kg/m^3.

3–22 Determine the pressure exerted on a diver at 30 m below the free surface of the sea. Assume a barometric pressure of 101 kPa and a specific gravity of 1.03 for seawater. *Answer:* 404.0 kPa

3–23E Determine the pressure exerted on the surface of a submarine cruising 300 ft below the free surface of the sea. Assume that the barometric pressure is 14.7 psia and the specific gravity of seawater is 1.03.

3–24 A gas is contained in a vertical, frictionless piston–cylinder device. The piston has a mass of 4 kg and a cross-sectional area of 35 cm^2. A compressed spring above the piston exerts a force of 60 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder. *Answer:* 123.4 kPa

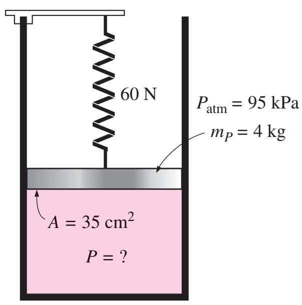

FIGURE P3–24

3–25 Reconsider Prob. 3–24. Using EES (or other) software, investigate the effect of the spring force in the range of 0 to 500 N on the pressure inside the cylinder. Plot the pressure against the spring force, and discuss the results.

3–26 Both a gage and a manometer are attached to a gas tank to measure its pressure. If the reading on the pressure gage is 80 kPa, determine the distance between the two fluid levels of the manometer if the fluid is (*a*) mercury (ρ = 13,600 kg/m^3) or (*b*) water (ρ = 1000 kg/m^3).

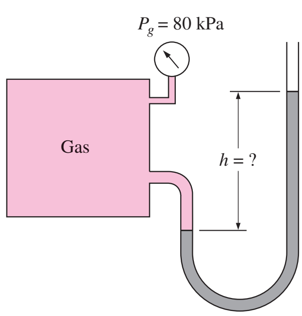

FIGURE P3–26

3–27 Reconsider Prob. 3–26. Using EES (or other) software, investigate the effect of the manometer fluid density in the range of 800 to 13,000 kg/m^3 on the differential fluid height of the manometer. Plot the differential fluid height against the density, and discuss the results.

3–28 A manometer containing oil (ρ = 850 kg/m^3) is attached to a tank filled with air. If the oil-level difference between the two columns is 45 cm and the atmospheric pressure is 98 kPa, determine the absolute pressure of the air in the tank. *Answer:* 101.75 kPa

3–29 A mercury manometer (ρ = 13,600 kg/m^3) is connected to an air duct to measure the pressure inside. The difference in the manometer levels is 15 mm, and the atmospheric pressure is 100 kPa. (*a*) Judging from Fig. P3–29, determine if the pressure in the duct is above or below the atmospheric pressure. (*b*) Determine the absolute pressure in the duct.

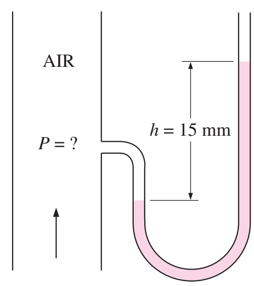

FIGURE P3–29

3–30 Repeat Prob. 3–29 for a differential mercury height of 30 mm.

3–31 Blood pressure is usually measured by wrapping a closed air-filled jacket equipped with a pressure gage around the upper arm of a person at the level of the heart. Using a mercury manometer and a stethoscope, the systolic pressure (the maximum pressure when the heart is pumping) and the diastolic pressure (the minimum pressure when the heart is resting) are measured in mmHg. The systolic and diastolic pressures of a healthy person are about 120 mmHg and 80 mmHg, respectively, and are indicated as 120/80. Express both of these gage pressures in kPa, psi, and meter water column.

3–32 The maximum blood pressure in the upper arm of a healthy person is about 120 mmHg. If a vertical tube open to the atmosphere is connected to the vein in the arm of the person, determine how high the blood will rise in the tube. Take the density of the blood to be 1050 kg/m^3.

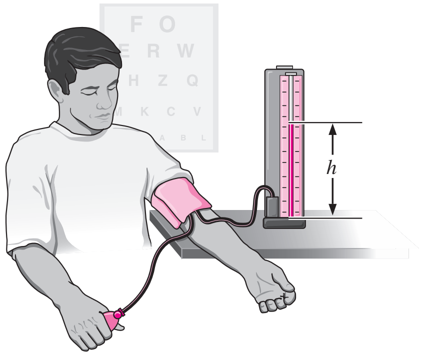

FIGURE P3–32

3–33 Consider a 1.8-m-tall man standing vertically in water and completely submerged in a pool. Determine the difference between the pressures acting at the head and at the toes of this man, in kPa.

3–34 Consider a U-tube whose arms are open to the atmosphere. Now water is poured into the U-tube from one arm, and light oil (ρ = 790 kg/m^3) from the other. One arm contains 70-cm-high water, while the other arm contains both fluids with an oil-to-water height ratio of 6. Determine the height of each fluid in that arm.

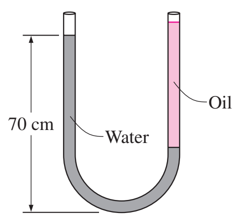

FIGURE P3–34

3–35 The hydraulic lift in a car repair shop has an output diameter of 30 cm and is to lift cars up to 2000 kg. Determine the fluid gage pressure that must be maintained in the reservoir.

3–36 Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in Fig. P3–36. Determine the pressure difference between the two pipelines. Take the density of seawater at that location to be ρ = 1035 kg/m^3. Can the air column be ignored in the analysis?

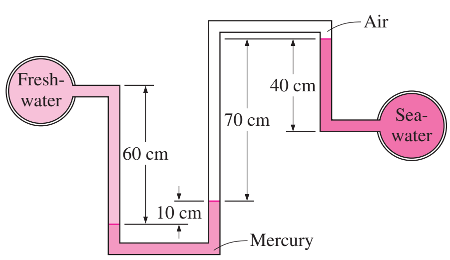

FIGURE P3–36

3–37 Repeat Prob. 3–36 by replacing the air with oil whose specific gravity is 0.72.

3–38E The pressure in a natural gas pipeline is measured by the manometer shown in Fig. P3–38E with one of the arms open to the atmosphere where the local atmospheric pressure is 14.2 psia. Determine the absolute pressure in the pipeline.

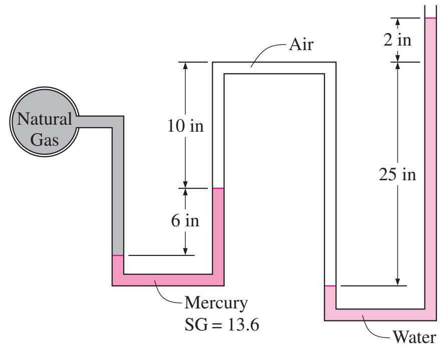

FIGURE P3–38E

3–39E Repeat Prob. 3–38E by replacing air by oil with a specific gravity of 0.69.

3–40 The gage pressure of the air in the tank shown in Fig. P3–40 is measured to be 65 kPa. Determine the differential height h of the mercury column.

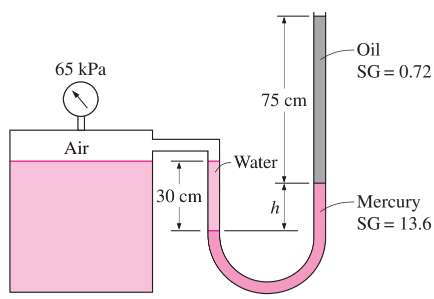

FIGURE P3–40

3–41 Repeat Prob. 3–40 for a gage pressure of 45 kPa.

3–42 The top part of a water tank is divided into two compartments, as shown in Fig. P3–42. Now a fluid with an unknown density is poured into one side, and the water level rises a certain amount on the other side to compensate for this effect. Based on the final fluid heights shown on the figure, determine the density of the fluid added. Assume the liquid does not mix with water.

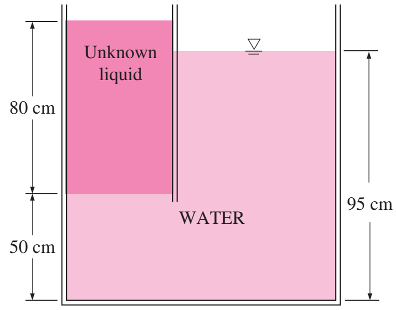

FIGURE P3–42

3–43 The 500-kg load on the hydraulic lift shown in Fig. P3–43 is to be raised by pouring oil (ρ = 780 kg/m^3) into a thin tube. Determine how high h should be in order to begin to raise the weight.

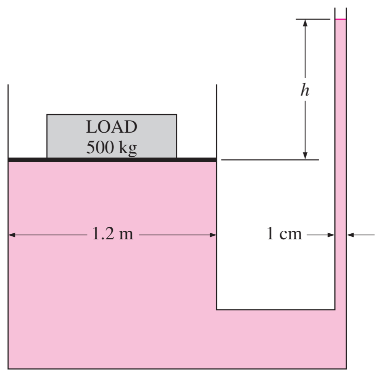

FIGURE P3–43

3–44E Two oil tanks are connected to each other through a manometer. If the difference between the mercury levels in the two arms is 32 in, determine the pressure difference between the two tanks. The densities of oil and mercury are 45 lbm/ft^3 and 848 lbm/ft^3, respectively.

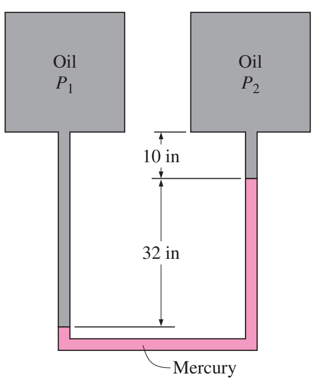

FIGURE P3–44E

3–45 Pressure is often given in terms of a liquid column and is expressed as "pressure head." Express the standard atmospheric pressure in terms of (*a*) mercury (SG = 13.6), (*b*) water (SG = 1.0), and (*c*) glycerin (SG = 1.26) columns. Explain why we usually use mercury in manometers.

3–46 A simple experiment has long been used to demonstrate how negative pressure prevents water from being spilled out of an inverted glass. A glass that is fully filled by water and covered with a thin paper is inverted, as shown in Fig. P3–46. Determine the pressure at the bottom of the glass, and explain why water does not fall out.

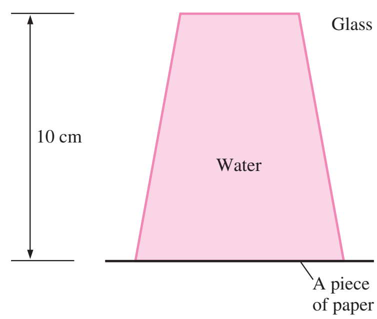

FIGURE P3–46

3–47 Two chambers with the same fluid at their base are separated by a piston whose weight is 25 N, as shown in Fig. P3–47. Calculate the gage pressures in chambers *A* and *B*.

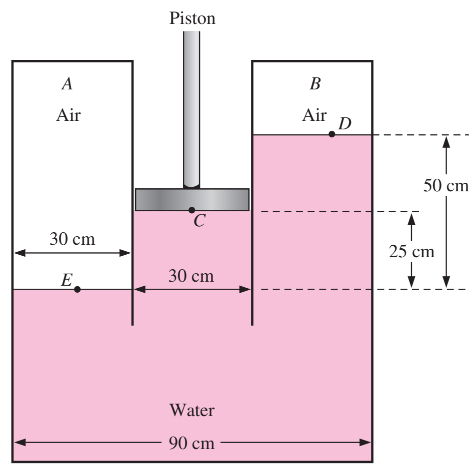

FIGURE P3–47

3–48 Consider a double-fluid manometer attached to an air pipe shown in Fig. P3–48. If the specific gravity of one fluid is 13.55, determine the specific gravity of the other fluid for the indicated absolute pressure of air. Take the atmospheric pressure to be 100 kPa. *Answer:* 5.0

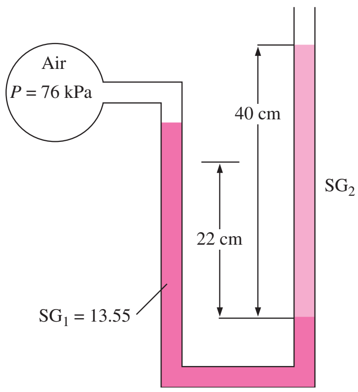

FIGURE P3–48

3–49 The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown in Fig. P3–49. For the given fluid heights and specific gravities, calculate the pressure difference $\Delta P = P_B - P_A$.

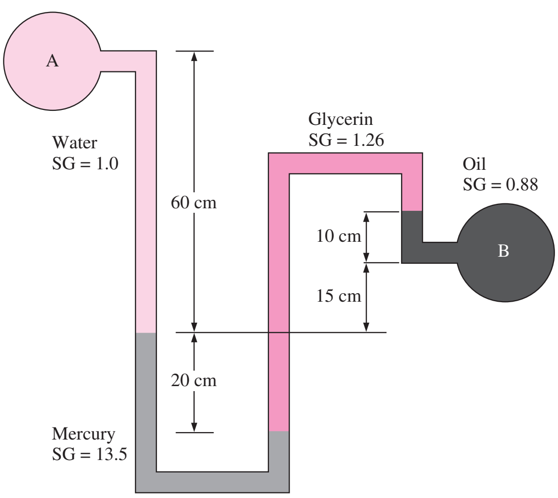

FIGURE P3–49

3–50 Consider the system shown in Fig. P3–50. If a change of 0.7 kPa in the pressure of air causes the brine-mercury interface in the right column to drop by 5 mm in the brine level in the right column while the pressure in the brine pipe remains constant, determine the ratio of A_2/A_1.

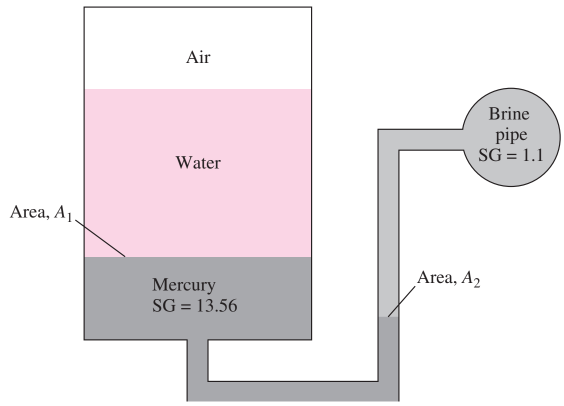

FIGURE P3–50

3–51 Two water tanks are connected to each other through a mercury manometer with inclined tubes, as shown in Fig. P3–51. If the pressure difference between the two tanks is 20 kPa, calculate a and θ.

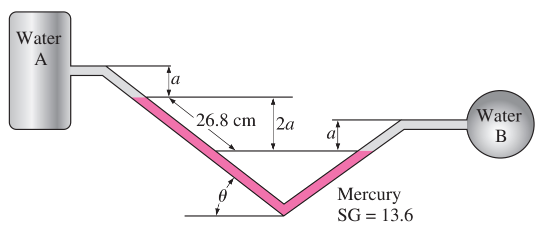

FIGURE P3–51

3–52 A multifluid container is connected to a U-tube, as shown in Fig. P3–52. For the given specific gravities and fluid column heights, determine the gage pressure at A. Also determine the height of a mercury column that would create the same pressure at A. *Answers:* 0.471 kPa, 0.353 cm

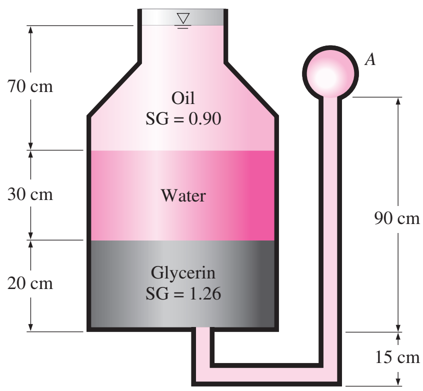

FIGURE P3–52

Fluid Statics: Hydrostatic Forces on Plane and Curved Surfaces

3–53C Define the resultant hydrostatic force acting on a submerged surface, and the center of pressure.

3–54C Someone claims that she can determine the magnitude of the hydrostatic force acting on a plane surface submerged in water regardless of its shape and orientation if she knew the vertical distance of the centroid of the surface from the free surface and the area of the surface. Is this a valid claim? Explain.

3–55C A submerged horizontal flat plate is suspended in water by a string attached at the centroid of its upper surface. Now the plate is rotated 45° about an axis that passes through its centroid. Discuss the change on the hydrostatic force acting on the top surface of this plate as a result of this rotation. Assume the plate remains submerged at all times.

3–56C You may have noticed that dams are much thicker at the bottom. Explain why dams are built that way.

3–57C Consider a submerged curved surface. Explain how you would determine the horizontal component of the hydrostatic force acting on this surface.

3–58C Consider a submerged curved surface. Explain how you would determine the vertical component of the hydrostatic force acting on this surface.

3–59C Consider a circular surface subjected to hydrostatic forces by a constant density liquid. If the magnitudes of the horizontal and vertical components of the resultant hydrostatic force are determined, explain how you would find the line of action of this force.

3–60 Consider a heavy car submerged in water in a lake with a flat bottom. The driver's side door of the car is 1.1 m high and 0.9 m wide, and the top edge of the door is 8 m below the water surface. Determine the net force acting on the door (normal to its surface) and the location of the pressure center if (*a*) the car is well-sealed and it contains air at atmospheric pressure and (*b*) the car is filled with water.

3–61E A long, solid cylinder of radius 2 ft hinged at point *A* is used as an automatic gate, as shown in Fig. P3–61E. When the water level reaches 15 ft, the cylindrical gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per ft length of the cylinder.

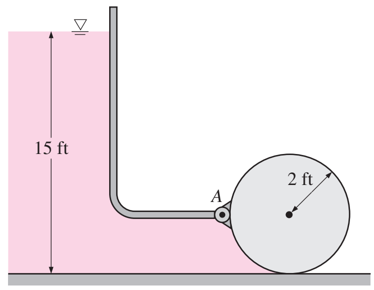

FIGURE P3–61E

3–62 Consider a 4-m-long, 4-m-wide, and 1.5-m-high aboveground swimming pool that is filled with water to the rim. (*a*) Determine the hydrostatic force on each wall and the distance of the line of action of this force from the ground. (*b*) If the height of the walls of the pool is doubled and the pool is filled, will the hydrostatic force on each wall double or quadruple? Why? *Answer:* (*a*) 44.1 kN

3–63E Consider a 200-ft-high, 1200-ft-wide dam filled to capacity. Determine (*a*) the hydrostatic force on the dam and (*b*) the force per unit area of the dam near the top and near the bottom.

3–64 A room in the lower level of a cruise ship has a 30-cm-diameter circular window. If the midpoint of the window is 5 m below the water surface, determine the hydrostatic force acting on the window, and the pressure center. Take the specific gravity of seawater to be 1.025. *Answers:* 3554 N, 5.001 m

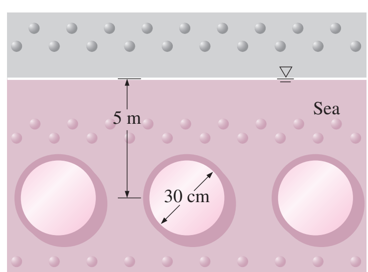

FIGURE P3–64

3–65 The water side of the wall of a 100-m-long dam is a quarter circle with a radius of 10 m. Determine the hydrostatic force on the dam and its line of action when the dam is filled to the rim.

3–66 A 4-m-high, 5-m-wide rectangular plate blocks the end of a 4-m-deep freshwater channel, as shown in Fig. P3–66. The plate is hinged about a horizontal axis along its upper edge through a point *A* and is restrained from opening by a fixed ridge at point *B*. Determine the force exerted on the plate by the ridge.

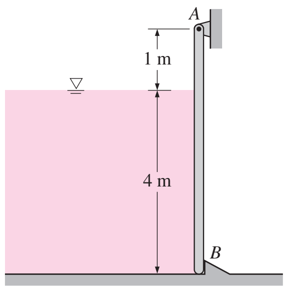

FIGURE P3–66

3–67 Reconsider Prob. 3–66. Using EES (or other) software, investigate the effect of water depth on the force exerted on the plate by the ridge. Let the water depth vary from 0 m to 5 m in increments of 0.5 m. Tabulate and plot your results.

3–68E The flow of water from a reservoir is controlled by a 5-ft-wide L-shaped gate hinged at point *A,* as shown in Fig. P3–68E. If it is desired that the gate open when the water height is 12 ft, determine the mass of the required weight *W*. *Answer:* 30,900 lbm

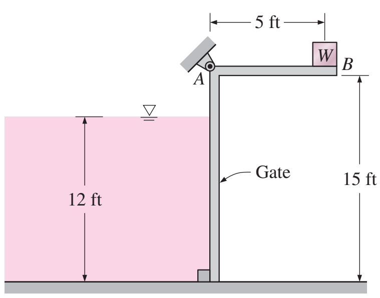

FIGURE P3–68E

3–69E Repeat Prob. 3–68E for a water height of 8 ft.

3–70 A water trough of semicircular cross section of radius 0.5 m consists of two symmetric parts hinged to each other at the bottom, as shown in Fig. P3–70. The two parts are held together by a cable and turnbuckle placed every 3 m along the length of the trough. Calculate the tension in each cable when the trough is filled to the rim.

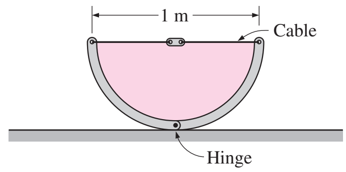

FIGURE P3–70

3–71 The two sides of a V-shaped water trough are hinged to each other at the bottom where they meet, as shown in Fig. P3–71, making an angle of 45° with the ground from both sides. Each side is 0.75 m wide, and the two parts are held together by a cable and turnbuckle placed every 6 m along the length of the trough. Calculate the tension in each cable when the trough is filled to the rim. *Answer:* 5510 N

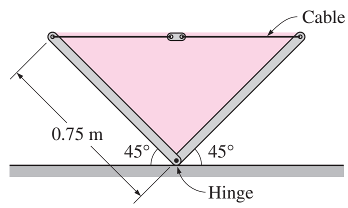

FIGURE P3–71

3–72 Repeat Prob. 3–71 for the case of a partially filled trough with a water height of 0.4 m directly above the hinge.

3–73 A retaining wall against a mud slide is to be constructed by placing 0.8-m-high and 0.2-m-wide rectangular concrete blocks (ρ = 2700 kg/m^3) side by side, as shown in Fig. P3–73. The friction coefficient between the ground and the concrete blocks is f = 0.3, and the density of the mud is about 1800 kg/m^3. There is concern that the concrete blocks may slide or tip over the lower left edge as the mud level rises. Determine the mud height at which (*a*) the blocks will

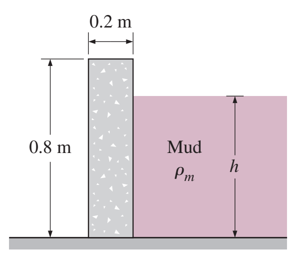

FIGURE P3–73

overcome friction and start sliding and (*b*) the blocks will tip over.

3–74 Repeat Prob. 3–73 for 0.4-m-wide concrete blocks.

3–75 A 4-m-long quarter-circular gate of radius 3 m and of negligible weight is hinged about its upper edge *A*, as shown in Fig. P3–75. The gate controls the flow of water over the ledge at *B*, where the gate is pressed by a spring. Determine the minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate.

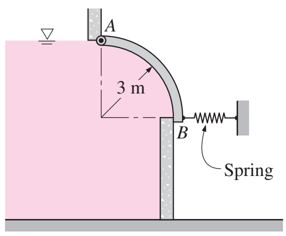

FIGURE P3–75

3–76 Repeat Prob. 3–75 for a radius of 4 m for the gate. *Answer:* 314 kN

Buoyancy

3–77C What is buoyant force? What causes it? What is the magnitude of the buoyant force acting on a submerged body whose volume is *V*? What are the direction and the line of action of the buoyant force?

3–78C Consider two identical spherical balls submerged in water at different depths. Will the buoyant forces acting on these two balls be the same or different? Explain.

3–79C Consider two 5-cm-diameter spherical balls—one made of aluminum, the other of iron—submerged in water. Will the buoyant forces acting on these two balls be the same or different? Explain.

3–80C Consider a 3-kg copper cube and a 3-kg copper ball submerged in a liquid. Will the buoyant forces acting on these two bodies be the same or different? Explain.

3–81C Discuss the stability of (*a*) a submerged and (*b*) a floating body whose center of gravity is above the center of buoyancy.

3–82 The density of a liquid is to be determined by an old 1-cm-diameter cylindrical hydrometer whose division marks are completely wiped out. The hydrometer is first dropped in water, and the water level is marked. The hydrometer is then dropped into the other liquid, and it is observed that the mark for water has risen 0.5 cm above the liquid–air interface. If the height of the water mark is 10 cm, determine the density of the liquid.

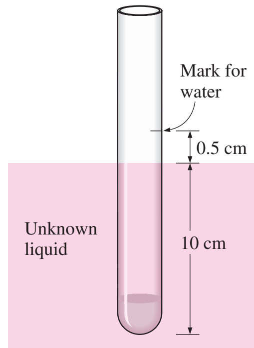

FIGURE P3–82

3–83E A crane is used to lower weights into a lake for an underwater construction project. Determine the tension in the rope of the crane due to a 3-ft-diameter spherical steel block (density = 494 lbm/ft^3) when it is (*a*) suspended in the air and (*b*) completely immersed in water.

3–84 The volume and the average density of an irregularly shaped body are to be determined by using a spring scale. The body weighs 7200 N in air and 4790 N in water. Determine the volume and the density of the body. State your assumptions.

3–85 Consider a large cubic ice block floating in seawater. The specific gravities of ice and seawater are 0.92 and 1.025, respectively. If a 10-cm-high portion of the ice block extends above the surface of the water, determine the height of the ice block below the surface. *Answer:* 87.6 cm

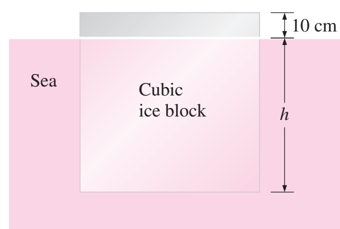

FIGURE P3–85

3–86 A 170-kg granite rock (ρ = 2700 kg/m^3) is dropped into a lake. A man dives in and tries to lift the rock. Determine how much force the man needs to apply to lift it from the bottom of the lake. Do you think he can do it?

3–87 It is said that Archimedes discovered his principle during a bath while thinking about how he could determine if King Hiero's crown was actually made of pure gold. While in the bathtub, he conceived the idea that he could determine the average density of an irregularly shaped object by weighing it in air and also in water. If the crown weighed 3.20 kgf (= 31.4 N) in air and 2.95 kgf (= 28.9 N) in water, determine if the crown is made of pure gold. The density of gold is 19,300 kg/m^3. Discuss how you can solve this problem without weighing the crown in water but by using an ordinary bucket with no calibration for volume. You may weigh anything in air.

3–88 One of the common procedures in fitness programs is to determine the fat-to-muscle ratio of the body. This is based on the principle that the muscle tissue is denser than the fat tissue, and, thus, the higher the average density of the body, the higher is the fraction of muscle tissue. The average density of the body can be determined by weighing the person in air and also while submerged in water in a tank. Treating all tissues and bones (other than fat) as muscle with an equivalent density of ρ_{muscle}, obtain a relation for the volume fraction of body fat x_{fat}. *Answer:* $x_{fat} = (\rho_{muscle} - \rho_{ave})/(\rho_{muscle} - \rho_{fat})$.

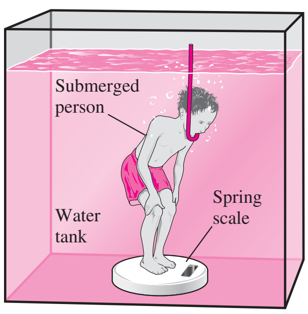

FIGURE P3–88E

3–89 The hull of a boat has a volume of 150 m^3, and the total mass of the boat when empty is 8560 kg. Determine how much load this boat can carry without sinking (*a*) in a lake and (*b*) in seawater with a specific gravity of 1.03.

Fluids in Rigid-Body Motion

3–90C Under what conditions can a moving body of fluid be treated as a rigid body?

3–91C Consider a glass of water. Compare the water pressures at the bottom surface for the following cases: the glass is (*a*) stationary, (*b*) moving up at constant velocity, (*c*) moving down at constant velocity, and (*d*) moving horizontally at constant velocity.

3–92C Consider two identical glasses of water, one stationary and the other moving on a horizontal plane with constant acceleration. Assuming no splashing or spilling occurs, which glass will have a higher pressure at the (*a*) front, (*b*) midpoint, and (*c*) back of the bottom surface?

3–93C Consider a vertical cylindrical container partially filled with water. Now the cylinder is rotated about its axis at a specified angular velocity, and rigid-body motion is established. Discuss how the pressure will be affected at the midpoint and at the edges of the bottom surface due to rotation.

3–94 A water tank is being towed by a truck on a level road, and the angle the free surface makes with the horizontal is measured to be 15°. Determine the acceleration of the truck.

3–95 Consider two water tanks filled with water. The first tank is 8 m high and is stationary, while the second tank is 2 m high and is moving upward with an acceleration of 5 m/s^2. Which tank will have a higher pressure at the bottom?

3–96 A water tank is being towed on an uphill road that makes 20° with the horizontal with a constant acceleration of 5 m/s^2 in the direction of motion. Determine the angle the free surface of water makes with the horizontal. What would your answer be if the direction of motion were downward on the same road with the same acceleration?

3–97E A 2-ft-diameter vertical cylindrical tank open to the atmosphere contains 1-ft-high water. The tank is now rotated about the centerline, and the water level drops at the center while it rises at the edges. Determine the angular velocity at which the bottom of the tank will first be exposed. Also determine the maximum water height at this moment.

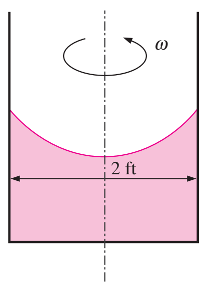

FIGURE P3–97E

3–98 A 60-cm-high, 40-cm-diameter cylindrical water tank is being transported on a level road. The highest acceleration anticipated is 4 m/s^2. Determine the allowable initial water height in the tank if no water is to spill out during acceleration. *Answer:* 51.8 cm

3–99 A 40-cm-diameter, 90-cm-high vertical cylindrical container is partially filled with 60-cm-high water. Now the cylinder is rotated at a constant angular speed of 120 rpm. Determine how much the liquid level at the center of the cylinder will drop as a result of this rotational motion.

3–100 A fish tank that contains 40-cm-high water is moved in the cabin of an elevator. Determine the pressure at the bottom of the tank when the elevator is (*a*) stationary, (*b*) moving up with an upward acceleration of 3 m/s^2, and (*c*) moving down with a downward acceleration of 3 m/s^2.

3–101 A 3-m-diameter vertical cylindrical milk tank rotates at a constant rate of 12 rpm. If the pressure at the center of the bottom surface is 130 kPa, determine the pressure at the edge of the bottom surface of the tank. Take the density of the milk to be 1030 kg/m.

3–102 Milk with a density of 1020 kg/m^3 is transported on a level road in a 7-m-long, 3-m-diameter cylindrical tanker. The tanker is completely filled with milk (no air space), and it accelerates at 2.5 m/s^2. If the minimum pressure in the tanker is 100 kPa, determine the maximum pressure and its location. *Answer:* 47.9 kPa

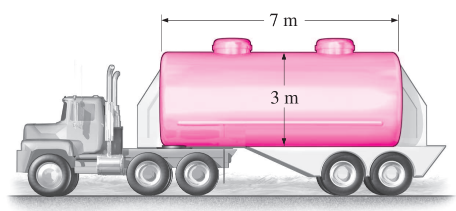

FIGURE P3–102

3–103 Repeat Prob. 3–102 for a deceleration of 2.5 m/s^2.

3–104 The distance between the centers of the two arms of a U-tube open to the atmosphere is 25 cm, and the U-tube contains 20-cm-high alcohol in both arms. Now the U-tube is rotated about the left arm at 4.2 rad/s. Determine the elevation difference between the fluid surfaces in the two arms.

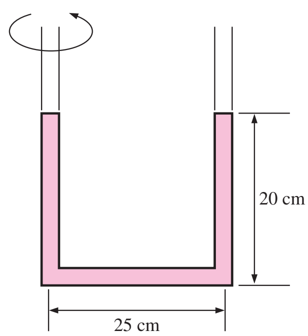

FIGURE P3–104

3–105 A 1.2-m-diameter, 3-m-high sealed vertical cylinder is completely filled with gasoline whose density is 740 kg/m^3. The tank is now rotated about its vertical axis at a rate of 70 rpm. Determine (*a*) the difference between the pressures at the centers of the bottom and top surfaces and (*b*) the difference between the pressures at the center and the edge of the bottom surface.

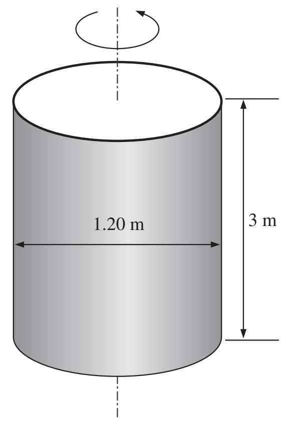

FIGURE P3–105

3–106 Reconsider Prob. 3–105. Using EES (or other) software, investigate the effect of rotational speed on the pressure difference between the center and the edge of the bottom surface of the cylinder. Let the rotational speed vary from 0 rpm to 500 rpm in increments of 50 rpm. Tabulate and plot your results.

3–107E A 20-ft-long, 8-ft-high rectangular tank open to the atmosphere is towed by a truck on a level road. The tank is filled with water to a depth of 6 ft. Determine the maximum acceleration or deceleration allowed if no water is to spill during towing.

3–108E An 8-ft-long tank open to the atmosphere initially contains 3-ft-high water. It is being towed by a truck on a level road. The truck driver applies the brakes and the water level at the front rises 0.5 ft above the initial level. Determine the deceleration of the truck. *Answer:* 4.08 ft/s^2

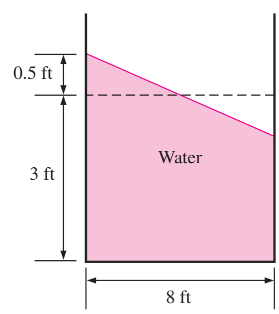

FIGURE P3–108E

3–109 A 3-m-diameter, 7-m-long cylindrical tank is completely filled with water. The tank is pulled by a truck on a level road with the 7-m-long axis being horizontal. Determine the pressure difference between the front and back ends of the tank along a horizontal line when the truck (*a*) accelerates at 3 m/s^2 and (*b*) decelerates at 4 m/s^2.

Review Problems

3–110 An air-conditioning system requires a 20-m-long section of 15-cm-diameter ductwork to be laid underwater. Determine the upward force the water will exert on the duct. Take the densities of air and water to be 1.3 kg/m^3 and 1000 kg/m^3, respectively.

3–111 Balloons are often filled with helium gas because it weighs only about one-seventh of what air weighs under identical conditions. The buoyancy force, which can be expressed as $F_b = \rho_{air} g V_{balloon}$, will push the balloon upward. If the balloon has a diameter of 10 m and carries two people, 70 kg each, determine the acceleration of the balloon when it is first released. Assume the density of air is $\rho = 1.16$ kg/m^3, and neglect the weight of the ropes and the cage. *Answer:* 16.5 m/s^2

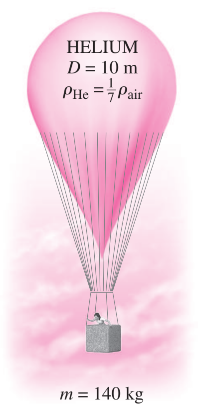

FIGURE P3–111

3–112 Reconsider Prob. 3–111. Using EES (or other) software, investigate the effect of the number of people carried in the balloon on acceleration. Plot the acceleration against the number of people, and discuss the results.

3–113 Determine the maximum amount of load, in kg, the balloon described in Prob. 3–111 can carry. *Answer:* 520.6 kg

3–114E The pressure in a steam boiler is given to be 75 kgf/cm^2. Express this pressure in psi, kPa, atm, and bars.

3–115 The basic barometer can be used as an altitude-measuring device in airplanes. The ground control reports a barometric reading of 753 mmHg while the pilot's reading is 690 mmHg. Estimate the altitude of the plane from ground level if the average air density is 1.20 kg/m^3. *Answer:* 714 m

3–116 The lower half of a 10-m-high cylindrical container is filled with water ($\rho = 1000$ kg/m^3) and the upper half with oil that has a specific gravity of 0.85. Determine the pressure difference between the top and bottom of the cylinder. *Answer:* 90.7 kPa

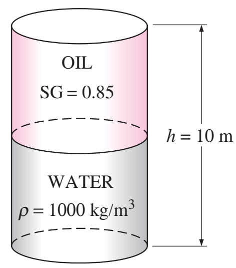

FIGURE P3–116

3–117 A vertical, frictionless piston–cylinder device contains a gas at 500 kPa. The atmospheric pressure outside is 100 kPa, and the piston area is 30 cm^2. Determine the mass of the piston.

3–118 A pressure cooker cooks a lot faster than an ordinary pan by maintaining a higher pressure and temperature inside. The lid of a pressure cooker is well sealed, and steam can escape only through an opening in the middle of the lid. A separate metal piece, the petcock, sits on top of this opening

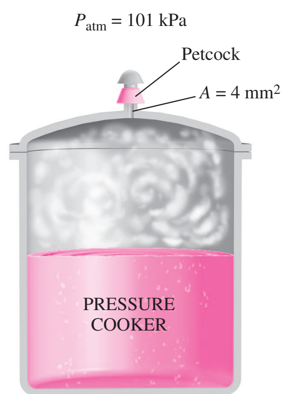

FIGURE P3–118

and prevents steam from escaping until the pressure force overcomes the weight of the petcock. The periodic escape of the steam in this manner prevents any potentially dangerous pressure buildup and keeps the pressure inside at a constant value. Determine the mass of the petcock of a pressure cooker whose operation pressure is 100 kPa gage and has an opening cross-sectional area of 4 mm^2. Assume an atmospheric pressure of 101 kPa, and draw the free-body diagram of the petcock. *Answer:* 40.8 g

3–119 A glass tube is attached to a water pipe, as shown in Fig. P3–119. If the water pressure at the bottom of the tube is 115 kPa and the local atmospheric pressure is 92 kPa, determine how high the water will rise in the tube, in m. Assume g = 9.8 m/s^2 at that location and take the density of water to be 1000 kg/m^3.

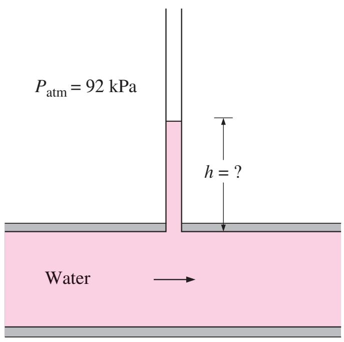

FIGURE P3–119

3–120 The average atmospheric pressure on earth is approximated as a function of altitude by the relation $P_{atm} = 101.325\,(1 - 0.02256z)^{5.256}$, where P_{atm} is the atmospheric pressure in kPa and z is the altitude in km with $z = 0$ at sea level. Determine the approximate atmospheric pressures at Atlanta (z = 306 m), Denver (z = 1610 m), Mexico City (z = 2309 m), and the top of Mount Everest (z = 8848 m).

3–121 When measuring small pressure differences with a manometer, often one arm of the manometer is inclined to improve the accuracy of reading. (The pressure difference is still proportional to the *vertical* distance and not the actual length of the fluid along the tube.) The air pressure in a circular duct is to be measured using a manometer whose open arm is inclined 35° from the horizontal, as shown in Fig. P3–121. The density of the liquid in the manometer is 0.81 kg/L, and the vertical distance between the fluid levels in the two arms of the manometer is 8 cm. Determine the gage pressure of air in the duct and the length of the fluid column in the inclined arm above the fluid level in the vertical arm.

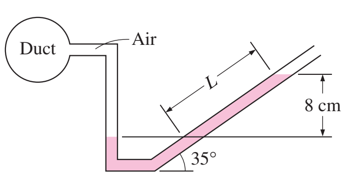

FIGURE P3–121

3–122E Consider a U-tube whose arms are open to the atmosphere. Now equal volumes of water and light oil (ρ = 49.3 lbm/ft^3) are poured from different arms. A person blows from the oil side of the U-tube until the contact surface of the two fluids moves to the bottom of the U-tube, and thus the liquid levels in the two arms are the same. If the fluid height in each arm is 30 in, determine the gage pressure the person exerts on the oil by blowing.

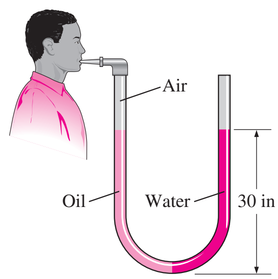

FIGURE P3–122E

3–123 Intravenous infusions are usually driven by gravity by hanging the fluid bottle at sufficient height to counteract the blood pressure in the vein and to force the fluid into the body. The higher the bottle is raised, the higher the flow rate of the fluid will be. (*a*) If it is observed that the fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, determine the gage pressure of the blood. (*b*) If the gage pressure of the fluid at the arm level needs to be 20 kPa for sufficient flow rate, determine how high the bottle must be placed. Take the density of the fluid to be 1020 kg/m^3.

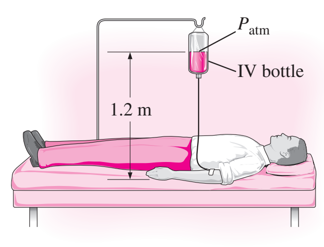

FIGURE P3–123

3–124 A gasoline line is connected to a pressure gage through a double-U manometer, as shown in Fig. P3–124. If the reading of the pressure gage is 370 kPa, determine the gage pressure of the gasoline line.

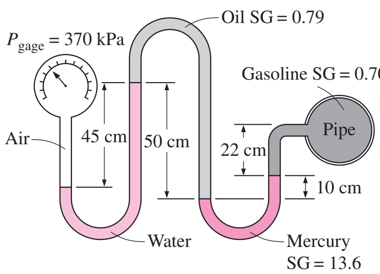

FIGURE P3–124

3–125 Repeat Prob. 3–124 for a pressure gage reading of 240 kPa.

3–126E A water pipe is connected to a double-U manometer as shown in Fig. P3–1026E at a location where the local atmospheric pressure is 14.2 psia. Determine the absolute pressure at the center of the pipe.

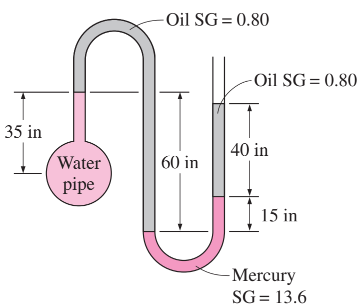

FIGURE P3–126E

3–127 The pressure of water flowing through a pipe is measured by the arrangement shown in Fig. P3–127. For the values given, calculate the pressure in the pipe.

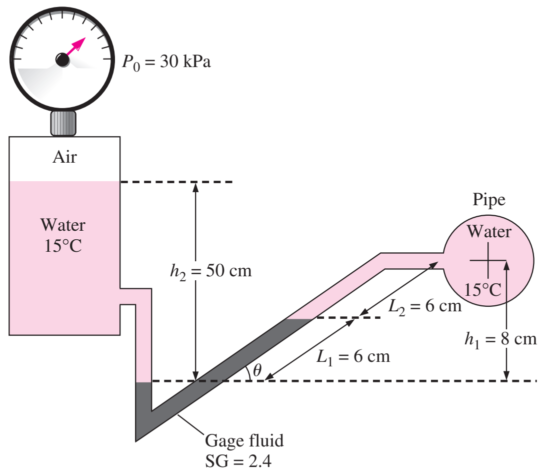

FIGURE P3–127

3–128 Consider a U-tube filled with mercury except the 18-cm-high portion at the top, as shown in Fig. P3–128. The diameter of the right arm of the U-tube is $D = 2$ cm, and the diameter of the left arm is twice that. Oil with a specific gravity of 2.72 is poured into the left arm, forcing some mercury from the left arm into the right one. Determine the maximum amount of oil that can be added into the left arm. *Answer:* 0.256 L

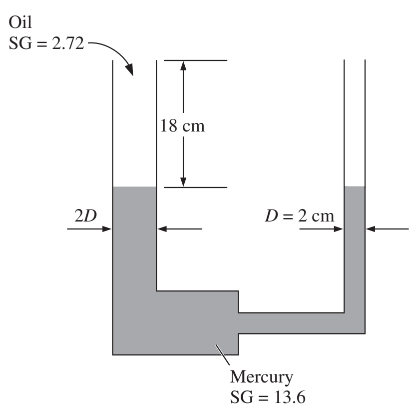

FIGURE P3–128

3–129 A teapot with a brewer at the top is used to brew tea, as shown in Fig. P3–129. The brewer may partially block the vapor from escaping, causing the pressure in the teapot to rise and an overflow from the service tube to occur. Disregarding thermal expansion and the variation in the amount of water in the service tube to be negligible relative to the amount of water in the teapot, determine the maximum cold-water height that would not cause an overflow at gage pressures of up to 0.32 kPa for the vapor.

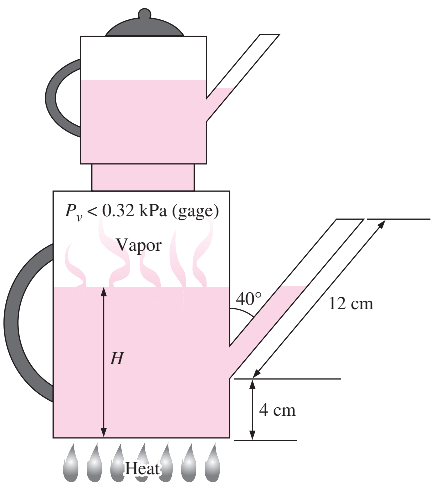

FIGURE P3–129

3–130 Repeat Prob. 3–129 by taking the thermal expansion of water into consideration as it is heated from 20°C to the boiling temperature of 100°C.

3–131 It is well known that the temperature of the atmosphere varies with altitude. In the troposphere, which extends to an altitude of 11 km, for example, the variation of temperature can be approximated by $T = T_0 - \beta z$, where T_0 is the temperature at sea level, which can be taken to be 288.15 K, and $\beta = 0.0065$ K/m. The gravitational acceleration also changes with altitude as $g(z) = g_0/(1 + z/6{,}370{,}320)^2$ where $g_0 = 9.807$ m/s^2 and z is the elevation from sea level in m. Obtain a relation for the variation of pressure in the troposphere (*a*) by ignoring and (*b*) by considering the variation of g with altitude.

3–132 The variation of pressure with density in a thick gas layer is given by $P = C\rho^n$, where C and n are constants. Noting that the pressure change across a differential fluid layer of thickness dz in the vertical z-direction is given as $dP = -\rho g\, dz$, obtain a relation for pressure as a function of elevation z. Take the pressure and density at $z = 0$ to be P_0 and ρ_0, respectively.

3–133 Pressure transducers are commonly used to measure pressure by generating analog signals usually in the range of 4 mA to 20 mA or 0 V-dc to 10 V-dc in response to applied pressure. The system whose schematic is shown in Fig. P3–133 can be used to calibrate pressure transducers. A rigid container is filled with pressurized air, and pressure is measured by the manometer attached. A valve is used to regulate the pressure in the container. Both the pressure and the electric signal are measured simultaneously for various settings, and the results are tabulated. For the given set of measurements, obtain the calibration curve in the form of $P = aI + b$, where a and b are constants, and calculate the pressure that corresponds to a signal of 10 mA.

Δh, mm	28.0	181.5	297.8	413.1	765.9
I, mA	4.21	5.78	6.97	8.15	11.76
Δh, mm	1027	1149	1362	1458	1536
I, mA	14.43	15.68	17.86	18.84	19.64

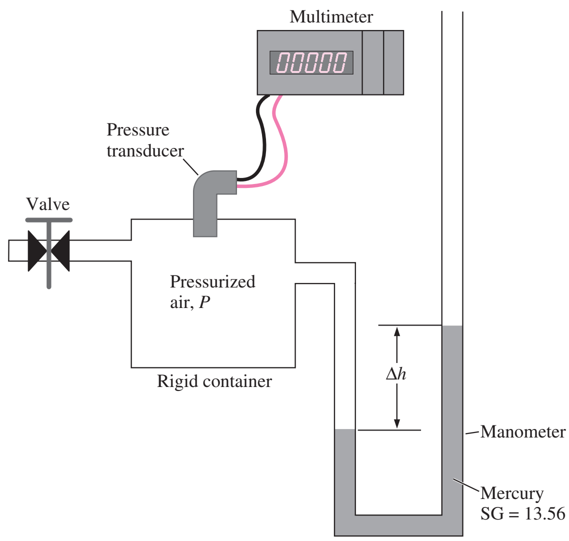

FIGURE P3–133

3–134 A system is equipped with two pressure gages and a manometer, as shown in Fig. P3–134. For Δh = 8 cm, determine the pressure difference $\Delta P = P_2 - P_1$.

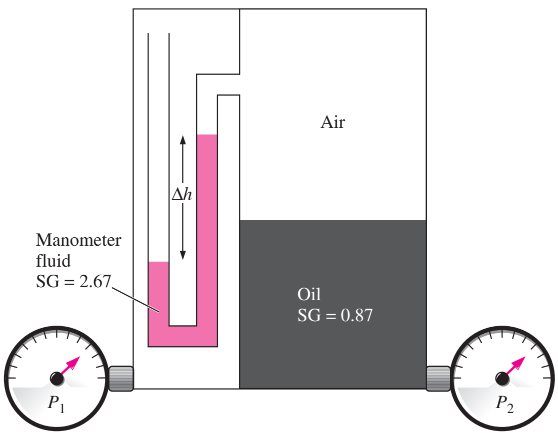

FIGURE P3–134

3–135 An oil pipeline and a 1.3-m^3 rigid air tank are connected to each other by a manometer, as shown in Fig. P3–135. If the tank contains 15 kg of air at 80°C, determine (*a*) the absolute pressure in the pipeline and (*b*) the change in Δh when the temperature in the tank drops to 20°C. Assume the pressure in the oil pipeline to remain constant, and the air volume in the manometer to be negligible relative to the volume of the tank.

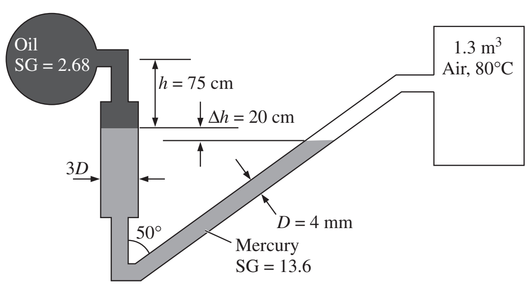

FIGURE P3–135

3–136 The density of a floating body can be determined by tying weights to the body until both the body and the weights are completely submerged, and then weighing them separately in air. Consider a wood log that weighs 1540 N in air. If it takes 34 kg of lead (ρ = 11,300 kg/m^3) to completely sink the log and the lead in water, determine the average density of the log. *Answer:* 835 kg/m^3

3–137 The 200-kg, 5-m-wide rectangular gate shown in Fig. P3–137 is hinged at *B* and leans against the floor at *A* making an angle of 45° with the horizontal. The gate is to be opened from its lower edge by applying a normal force at its center. Determine the minimum force *F* required to open the water gate. *Answer:* 520 kN

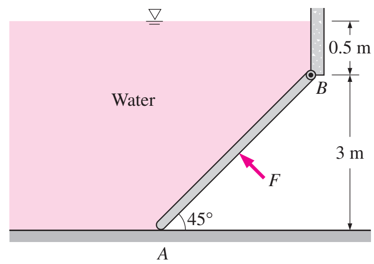

FIGURE P3–137

3–138 Repeat Prob. 3–137 for a water height of 1.2 m above the hinge at *B*.

3–139 A 3-m-high, 6-m-wide rectangular gate is hinged at the top edge at *A* and is restrained by a fixed ridge at *B*. Determine the hydrostatic force exerted on the gate by the 5-m-high water and the location of the pressure center.

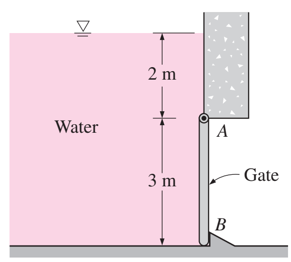

FIGURE P3–139

3–140 Repeat Prob. 3–139 for a total water height of 2 m.

3–141E A semicircular 30-ft-diameter tunnel is to be built under a 150-ft-deep, 800-ft-long lake, as shown in Fig. P3–141E. Determine the total hydrostatic force acting on the roof of the tunnel.

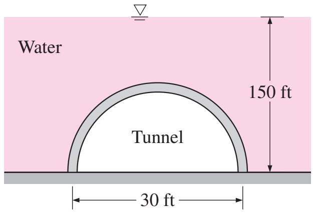

FIGURE P3–141E

3–142 A 50-ton, 6-m-diameter hemispherical dome on a level surface is filled with water, as shown in Fig. P3–142. Someone claims that he can lift this dome by making use of Pascal's law by attaching a long tube to the top and filling it with water. Determine the required height of water in the tube to lift the dome. Disregard the weight of the tube and the water in it. *Answer:* 0.77 m

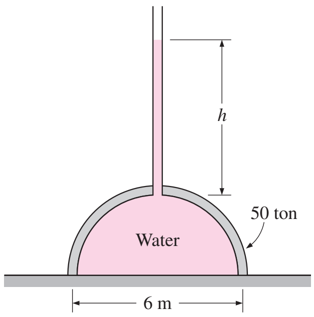

FIGURE P3–142

3–143 The water in a 25-m-deep reservoir is kept inside by a 150-m-wide wall whose cross section is an equilateral triangle, as shown in Fig. P3–143. Determine (*a*) the total force (hydrostatic + atmospheric) acting on the inner surface of the wall and its line of action and (*b*) the magnitude of the horizontal component of this force. Take P_{atm} = 100 kPa.

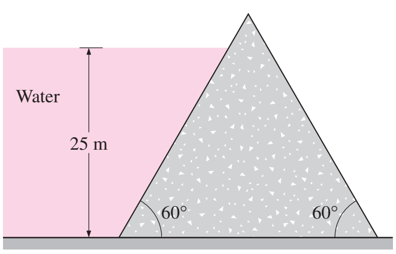

FIGURE P3–143

3–144 A U-tube contains water in the right arm, and another liquid in the left arm. It is observed that when the U-tube rotates at 30 rpm about an axis that is 15 cm from the right arm and 5 cm from the left arm, the liquid levels in both arms become the same. Determine the density of the fluid in the left arm.

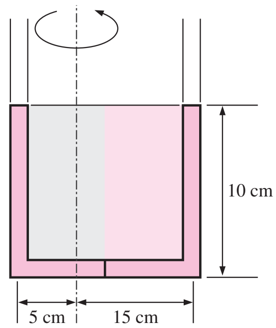

FIGURE P3–144

3–145 A 1-m-diameter, 2-m-high vertical cylinder is completely filled with gasoline whose density is 740 kg/m^3. The tank is now rotated about its vertical axis at a rate of 90 rpm, while being accelerated upward at 5 m/s^2. Determine (*a*) the difference between the pressures at the centers of the bottom and top surfaces and (*b*) the difference between the pressures at the center and the edge of the bottom surface.

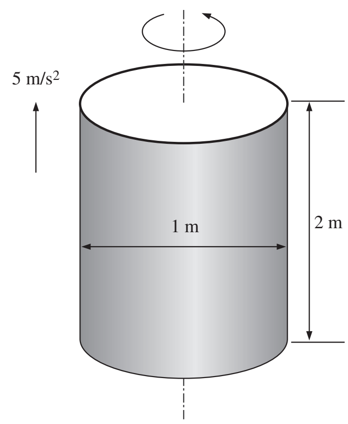

FIGURE P3–145

3–146 A 5-m-long, 4-m-high tank contains 2.5-m-deep water when not in motion and is open to the atmosphere through a vent in the middle. The tank is now accelerated to the right on a level surface at 2 m/s^2. Determine the maximum pressure in the tank relative to the atmospheric pressure. *Answer:* 29.5 kPa

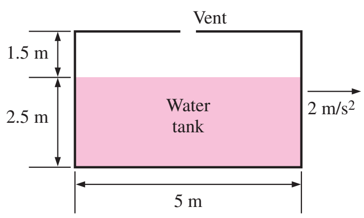

FIGURE P3–146

3–147 Reconsider Prob. 3–146. Using EES (or other) software, investigate the effect of acceleration on the slope of the free surface of water in the tank. Let the acceleration vary from 0 m/s^2 to 5 m/s^2 in increments of 0.5 m/s^2. Tabulate and plot your results.

3–148 An elastic air balloon having a diameter of 30 cm is attached to the base of a container partially filled with water at +4°C, as shown in Fig. P3–148. If the pressure of air above water is gradually increased from 100 kPa to 1.6 MPa, will the force on the cable change? If so, what is the percent change in the force? Assume the pressure on the free surface and the diameter of the balloon are related by $P = CD^n$, where C is a constant and $n = -2$. The weight of the balloon and the air in it is negligible. *Answer:* 98.4 percent

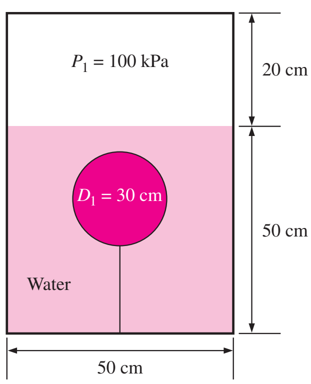

FIGURE P3–148

3–149 Reconsider Prob. 3–148. Using EES (or other) software, investigate the effect of air pressure above water on the cable force. Let this pressure vary from 0.1 MPa to 10 MPa. Plot the cable force versus the air pressure.

3–150 The average density of icebergs is about 917 kg/m^3. (*a*) Determine the percentage of the total volume of an iceberg submerged in seawater of density 1042 kg/m^3. (*b*) Although icebergs are mostly submerged, they are observed to turn over. Explain how this can happen. (Hint: Consider the temperatures of icebergs and seawater.)

3–151 A cylindrical container whose weight is 79 N is inverted and pressed into the water, as shown in Fig. P3–151. Determine the differential height h of the manometer and the force F needed to hold the container at the position shown.

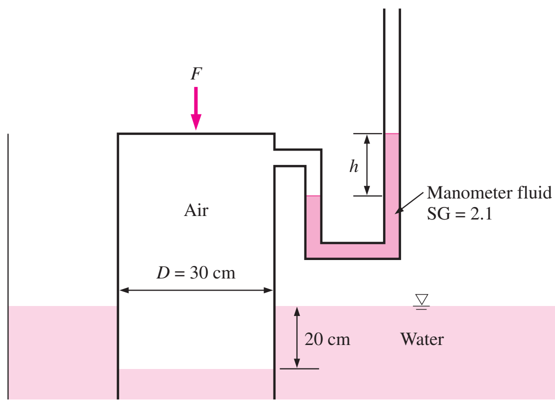

FIGURE P3–151

Design and Essay Problems

3–152 Shoes are to be designed to enable people of up to 80 kg to walk on freshwater or seawater. The shoes are to be made of blown plastic in the shape of a sphere, a (American) football, or a loaf of French bread. Determine the equivalent diameter of each shoe and comment on the proposed shapes from the stability point of view. What is your assessment of the marketability of these shoes?

3–153 The volume of a rock is to be determined without using any volume measurement devices. Explain how you would do this with a waterproof spring scale.