

Chapter 7: Kinetic Energy and Work

In this chapter, we will study the important concepts of kinetic energy and the closely related concept of work and power.

A- Kinetic Energy

Kinetic energy is a physical quantity, which is associated with the moving objects and defined as:

$$K = \frac{1}{2} mv^2$$

If the body is stationary ($v=0$), its kinetic energy is zero. The SI unit of kinetic energy is $\text{kg.m}^2/\text{s}^2$ or Joule (J), where $1 \text{ J} = 1 \text{ kg.m}^2/\text{s}^2$. Kinetic energy is a scalar quantity.

B- Work

The work is defined as the ability to perform a force along a certain displacement. There are different types of work as follows:

1- Work done by a constant force

The work done by the constant force \mathbf{F} is given by the scalar product of the force \mathbf{F} and the displacement \mathbf{d} .

$$W_F = \mathbf{F} \cdot \mathbf{d} = Fd \cos\theta$$

where θ is the angle between the force and displacement. The above equation means that the work is the product of the displacement magnitude by component of the force parallel to the displacement. Therefore, work is a scalar quantity (only magnitude, no direction) and can be positive, negative, or zero. The SI unit of work is (N.m) or joule (J) where $1 \text{ N.m} = 1 \text{ J}$.

Special cases and remarks:

- *If the angle between the force and displacement is zero (parallel), the work is*

$$W_F = F d \quad (\text{maximum work})$$

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- If the angle between the force and displacement is 90, the work is

$$W_F = 0$$

- If the angle between the force and displacement is 180 (anti-parallel), the work is

$$W_F = -F d \quad (\text{minimum work})$$

Positive work: the work done **by** a system is positive.

Negative work: the work done **on** a system is negative.

2- Work done by the gravity force

The work performed by the gravity force ($F=mg$) is given by

$$W_g = \mathbf{F} \cdot \mathbf{d} = mg \cdot d = mgd \cos\theta$$

Special cases and remarks:

- If the object is moving down (the angle between the force and displacement is zero), the work is

$$\boxed{W_g = mg d} \quad (\text{maximum work})$$

- If the object is moving up (the angle between the force and displacement is 180), the work is

$$\boxed{W_g = -mg d} \quad (\text{minimum work})$$

- The position where $W_g = 0$ is arbitrary.
- W_g is a function of **position only**. (It depends only on the **relative positions** of the earth and the block.)
- The work W_g depends **only** on the initial and final heights, **NOT** on the path.

3- Work done by the normal force

The work performed by the normal force (reaction force) is always zero, because the angle between the normal force and displacement is 90.

$$\boxed{W_N = 0}$$

Rule: Forces perpendicular to the motion do no work!

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4- Work done by the friction force

The work performed by the frictional force is always negative (at least in this stage), because the angle between the frictional force and displacement is 180.

$$W_f = -f_k d$$

5- Work done by the spring force

The work performed by a spring force is given by the following expression:

$$W_s = \frac{1}{2} kx^2$$

6- Net Work (the total work)

The total work is the summation of all types of work done on an object.

$$W_T = W_F + W_g + W_f$$

The total work done on an object determines its motion. If the total work is zero, the object moves at constant speed or is at rest. The object accelerates if the total work has a value other than zero.

C- Kinetic Energy and Work

The work can be defined as the change in the kinetic energy. If a body changes its velocity from v_1 to v_2 , the work done is defined as:

$$W = \Delta K = K_2 - K_1 = \frac{1}{2} m (v_2^2 - v_1^2)$$

This is known as the **work-energy theorem**: *The change of kinetic energy of a body is equal to the work of all the forces acting upon it.*

D- Mechanical Power

Power is defined as the rate of doing work or rate of using energy. It is defined as

$$P = W/t$$

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It is also defined as

$$P = \mathbf{F} \cdot \mathbf{v}$$

The SI unit of power is J/s, N.m/s, or Watt (W), where $1 \text{ W} = 1 \text{ J/s} = 1 \text{ N.m/s}$. Occasionally it is expressed by the mechanical horsepower (hp) where $1 \text{ hp} = 746 \text{ W}$. However power is a scalar quantity.

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Examples:

1. A 5 kg object moving at a speed of 6 m/s. Calculate its kinetic energy.

Solution

The kinetic energy is defined as

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times 6^2 = 90 \text{ J}$$

2. A car of mass 1000 kg accelerates at 2 m/s^2 for 10 s from an initial speed of 5 m/s. Determine the work done by the car.

Solution

The work is defined as the change in the kinetic energy. Therefore

$$W = \Delta K = K_2 - K_1 = \frac{1}{2} m (v_2^2 - v_1^2)$$

We know the initial speed ($v_1 = 5 \text{ m/s}$) but we have to find out the final speed (v_2) using the equation of motion. We know that

$$v_2 = v_1 + at$$

Therefore

$$v_2 = 5 + 2 \times 10 = 25 \text{ m/s}$$

Hence the work done by the car is

$$W = \frac{1}{2} \times 1000 \times (25^2 - 5^2) = 3 \times 10^5 \text{ J}$$

3. A force, $\mathbf{F} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ (N) is applied on a box. If the displacement of the box due to the force is $\mathbf{d} = \mathbf{i} + 3\mathbf{k}$ (m), find the work done.

Solution

The work done by the force is the scalar product of the force and its

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displacement, such as

$$W_F = \mathbf{F} \cdot \mathbf{d} = (2 \times 1) + (-3 \times 0) + (1 \times 3) = 5 \text{ J}$$

4. A box is pulled across the floor by a horizontal force of magnitude 80 N. How much work does the force exert to pull the object 5 m?

Solution

The work done by the force is

$$W_F = F d \cos\theta$$

But the force and displacement are parallel; the angle between them is zero. Therefore

$$W_F = F d = 80 \times 5 = 400 \text{ J}$$

5. A box is dragged across the floor by a force of magnitude 50 N directed 60° above the horizontal. Calculate the work done by the force to pull the object 6 m?

Solution

The work done by the force is

$$W_F = F d \cos\theta = 50 \times 6 \times \cos(60) = 150 \text{ J}$$

6. A horizontal force F is applied to move a 5 kg carton across the floor. If the acceleration of the carton is measured to be 2 m/s², how much work does F do in moving the carton 7 m?

Solution

While the force and displacement are parallel, the work done is

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$$W_F = F d$$

The force can be determined from Newton's second law as

$$F = ma = 5 \times 2 = 10 \text{ N}$$

Therefore the work is

$$W_F = F d = 10 \times 7 = 70 \text{ J}$$

7. A horizontal force **F** is applied to move a 6 kg carton across the floor. If the carton starts from rest and its speed after 3 sec is 6 m/s, how much work does **F** do in moving the carton 4 m?

Solution

While the force and displacement are parallel, the work done is

$$W_F = F d$$

However the force, **F**, can be determined from Newton's second law as

$$F = ma$$

The acceleration can be found using the motion equation

$$v = v_0 + at$$

Since the carton starts its motion from rest ($v_0=0$), the acceleration is

$$a = v/t = 6/3 = 2 \text{ m/s}^2$$

Therefore the force is

$$F = ma = 6 \times 2 = 12 \text{ N}$$

Hence the work done by the force **F** is

$$W_F = F d = 12 \times 4 = 48 \text{ J}$$

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8. A 5.0-kg box is raised a distance of 2.5 m from rest by a vertical applied force of 90 N. Find (a) the work done on the box by the applied force, and (b) the work done on the box by gravity.

Solution

(a) Using the definition of work in which the force and displacement are in the same direction (parallel), we get

$$W_F = F d = 90 \times 2.5 = 225 \text{ J}$$

(b) Using the definition of work done by gravity for moving up object (the force and displacement are anti-parallel), we get

$$W_g = -mg d = -5 \times 9.8 \times 2.5 = -122.5 \text{ J}$$

9. A box rests on a horizontal, frictionless surface. Ali pushes on the box with a force of 18 N to the right and Dima pushes on the box with a force of 12 N to the left. The box moves 4.0 m to the right. Find the work done by (a) Ali, (b) Dima, and (c) the net force.

Solution

(a) While the force applied by Ali is in the same direction of displacement (their angle is zero), the work done by Ali is

$$W_{\text{Ali}} = F_{\text{Ali}} d = 18 \times 4 = 72 \text{ J}$$

(b) While the force applied by Dima is in the opposite direction of displacement (their angle is 180), the work done by Dima is

$$W_{\text{Dima}} = -F_{\text{Dima}} d = -12 \times 4 = -48 \text{ J}$$

(c) The net force is

$$F_{\text{net}} = F_{\text{Ali}} + F_{\text{Dima}} = 18 - 12 = 6 \text{ N}$$

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We can also calculate the net force using the work concept where the net work is

$$W_{\text{net}} = F_{\text{net}} d$$

But the total (net) work is

$$W_{\text{net}} = W_{\text{Ali}} + W_{\text{Dima}} = 72 - 48 = 24 \text{ J}$$

Therefore we get

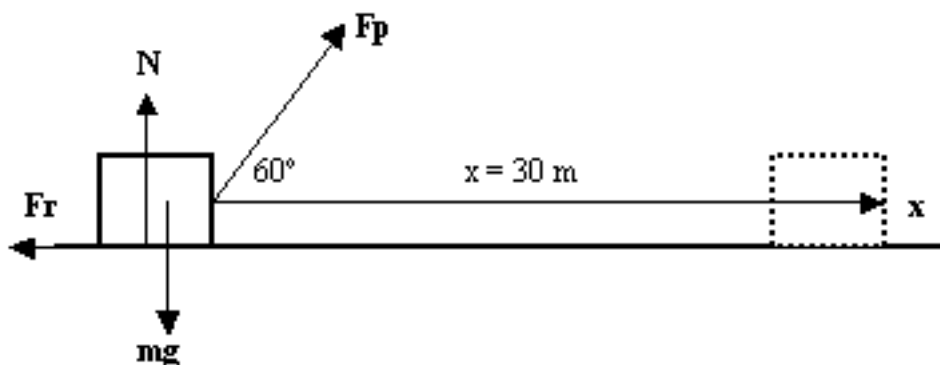
$$F_{\text{net}} = W_{\text{net}} / d = 24/4 = 6 \text{ N}$$

10. A 40 kg box is pulled 30 m on a horizontal floor by applying a force (F_p) of magnitude 100 N directed by an angle of 60° above the horizontal. If the floor exerts a friction force (f_r) of magnitude 20 N, calculate the work done by each one of these forces. Calculate the work done by the weight and the normal force. Calculate also the total work done on the box.

Solution

(a) The work done by the force is

$$W_F = F_p d \cos\theta = 100 \times 30 \times \cos 60 = 1500 \text{ J}$$



(b) The work done by the friction force is

$$W_f = f_r d \cos\theta = 20 \times 30 \times \cos 180 = -600 \text{ J}$$

Note that the angle between f_r and displacement is 180° because they point

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in opposite directions.

(c) The work done by the gravity force is

$$W_g = 0 \text{ J}$$

Note that the box is moving along x-axis and there is no displacement along the vertical direction, therefore its work is zero.

(c) The work done by the normal force is

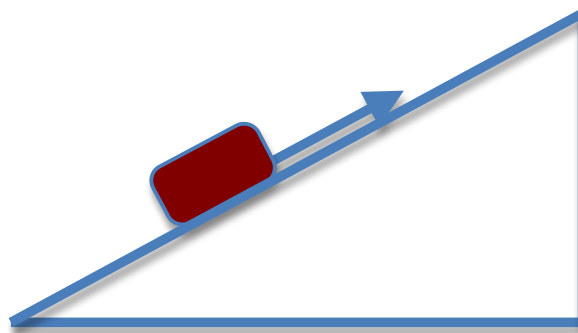
$$W_N = 0 \text{ J}$$

The normal force does no work because it is normal (perpendicular) to the displacement.

(d) The total work done on the box is

$$W_T = W_F + W_f + W_g + W_N = 1500 - 600 + 0 + 0 = 900 \text{ J}$$

11. A stationary block of 10 kg is pulled up along a smooth incline of length 10 m and height 5 m by applying an external force of 50 N parallel to the incline. At the end of incline, find (i) work done by the external force, (ii) work done by gravity, and (iii) speed of the block.



Solution

(i) The work done by the force is

$$W_F = F d \cos\theta = 50 \times 10 \times \cos 0 = 500 \text{ J}$$

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Note that both the force and displacement are in the same direction.

(ii) The work done by the gravity force is

$$W_g = F_g d \cos\theta = F_g d \cos\theta = 2 \times 9.8 \times 5 \times \cos 180 = -98 \text{ J}$$

(iii) The total work done is

$$W_T = W_F + W_g = 500 - 98 = 402 \text{ J}$$

Therefore the change in the kinetic energy is

$$W = \Delta K = K_2 - K_1 = \frac{1}{2} m v_2^2$$

Where the initial speed is zero. Hence the final speed is

$$v_2^2 = 2\Delta K/m = 2 \times 402/10 = 80.4 \text{ (m/s)}^2$$

$$v_2 = 9 \text{ m/s}$$

12. Maha slides a 1-kg toy for 3 m along the floor to her sister Reem. The toy is moving at 4 m/s when Maha lets go, and at 2 m/s when Reem catches it. (a) Find the work done by friction and (b) calculate the force of friction.

Solution

(a) We know that the total work done on a body equals the change in its kinetic energy. Since we have only a friction force, then the total work is due to it. Hence, we have

$$W_f = \Delta K = K_2 - K_1 = \frac{1}{2} m (v_2^2 - v_1^2) = \frac{1}{2} \times 1 \times (2^2 - 4^2) = -6 \text{ J}$$

(b) It is well known that the work done by the friction force is given by

$$W_f = -f_k d$$

Therefore the friction force is

$$f_k = -W_f/d = -(-6)/3 = 2 \text{ N}$$

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13. An 8.0 kg block initially at rest is pulled to the right for 3.0 m with a force of 12 N over a surface. Determine its final speed if: (a) the surface has no friction and (b) the surface has a coefficient of kinetic friction of 0.15

Solution

(a) In the case of frictionless surface (no friction force), the total work of the system is only the work done by the external force. We have

$$W_T = W_F = F d = 12 \times 3 = 36 \text{ J}$$

Therefore the change in kinetic energy is equivalent to the total work. Hence the final speed of the block is

$$W_T = \Delta K = K_2 - K_1 = \frac{1}{2} m (v_2^2 - v_1^2)$$

But the block starts from rest; therefore its initial speed is zero. The final speed is then

$$W_T = \frac{1}{2} m v_2^2$$

Or

$$v_2^2 = 2 W_T / m = 2 \times 36 / 8.0 = 9 \text{ (m/s)}^2$$

Or

$$v_2 = 3 \text{ m/s}$$

(b) In the case of frictional surface (there is friction force), the total work of the system is the work done by the external force and friction force. We have

$$W_T = W_F + W_f = F d - f_k d$$

The friction force is defined (from the previous chapter) as:

$$f_k = \mu_k N = \mu_k mg = 0.15 \times 8 \times 9.8 = 11.76 \text{ N}$$

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The total work, then, is

$$W_T = F d - f_k d$$

That gives

$$W_T = 12 \times 3 - 11.76 \times 3 = 0.72 \text{ J}$$

The final speed can be determined using the work-energy theorem as

$$W_T = \frac{1}{2} m v_2^2$$

Or

$$v_2^2 = 2 W_T / m = 2 \times 0.72 / 8.0 = 0.18 \text{ (m/s)}^2$$

Or

$$v_2 = 0.42 \text{ m/s}$$

14. A block of mass 1.6 kg resting on a frictionless surface is attached to a horizontal spring with a spring constant $k = 1.0 \times 10^3 \text{ N/m}$. The spring is compressed to 2.0 cm and released from rest. Find the velocity of the block when released.

Solution

We know that the work done by a spring is given by

$$W_s = \frac{1}{2} k x^2 = \frac{1}{2} \times 1.0 \times 10^3 \times (0.02)^2 = 0.2 \text{ J}$$

The total work is (there is only one kind of work due to the spring)

$$W_T = W_s = 0.2 \text{ J}$$

Therefore the final speed is

$$W_T = \frac{1}{2} m v^2$$

That will give us the following value

$$v = 0.5 \text{ m/s}$$

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15. A 5 kg ball is freely dropped from a height of 20 m above the ground. Calculate its speed when it hits the ground.

Solution

We know that the work done by the gravity is given by

$$W_g = mgd = 5 \times 9.8 \times 20 = 980 \text{ J}$$

The total work is (there is only gravity work)

$$W_T = W_g = 980 \text{ J}$$

Using the work-energy theorem, we get

$$W_T = \frac{1}{2} m v^2$$

Or

$$v = (2W_T/m)^{1/2} = (2 \times 980/5)^{1/2} = 19.8 \text{ m/s}$$

16. Zain does 240 J of work in pushing a box. If he does the work in 4 s, what is his power output?

Solution

The output power is defined as

$$P = W/t = 240/4 = 60 \text{ W}$$

17. Ahmad pushes a box along a smooth floor using a force of 210 N and a power output of 350 W. How long does it take Ahmad to push the box 20 m?

Solution

The output power is defined as

$$P = W/t$$

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Therefore the time taken for this operation is

$$t = W/P$$

But the work is

$$W = F d = 210 \times 20 = 4200 \text{ J}$$

Hence the time is

$$t = W/P = 4200 / 350 = 12 \text{ s}$$

17. A box is lifted at a constant speed of 7.0 m/s by a machine. If the power of the machine is 21000 W, find the force applied.

Solution

The output power is defined as

$$P = \mathbf{F} \cdot \mathbf{v}$$

But the force and velocity are parallel (both upwards), the force is

$$F = P/ v = 21000/7.0 = 3000 \text{ N}$$