FINAL EXAM

INTRODUCTION TO ELECTRODYNAMICS I (PHYS 331)

Fall 2010-2011

Student's **N**ame: ....

Student's **N**umber: ....

Question	Mark	
A1	5	
A2	5	
A3	6	
A4	6	
B1	6	
B2	6	
B3 or B4	6	
Total		

Del operator in Spherical Coordinates

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \emptyset}\hat{\emptyset}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\, sin\theta}\frac{\partial}{\partial \theta}(sin\theta\, A_\theta) + \frac{1}{r\, sin\theta}\frac{\partial A_\emptyset}{\partial \emptyset}$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r\, sin\theta}\left[\frac{\partial}{\partial \theta}(sin\theta\, A_\emptyset) - \frac{\partial A_\theta}{\partial \emptyset}\right]\hat{r} + \frac{1}{r\, sin\theta}\left[\frac{\partial A_r}{\partial \emptyset} - sin\theta\,\frac{\partial}{\partial r}(r\, A_\emptyset)\right]\hat{\theta} + + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\, A_\emptyset) - \frac{\partial A_r}{\partial \theta}\right]\hat{\emptyset}$$

Laplacian operator in Spherical Coordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\, sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\,\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \emptyset^2}$$

Laplacian operator in Cylindrical Coordinates

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$$

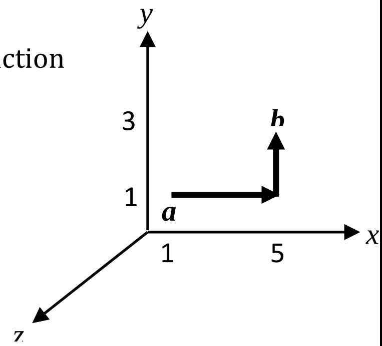
PART [A]: SOLVE ALL THESE QUESTIONS.

A1 [5 MARKS] You are given vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{b} = 3\hat{\imath} - 4\hat{\jmath}$. A third vector $\vec{c}$ in the *xy*-plane is normal to $\vec{a}$ and its scalar product with $\vec{b}$ is 11. **Find the vector $\vec{c}$.**

A2 [5 MARKS] **Evaluate the line integral** of the vector function

$$\vec{A}(x, y, z) = xy\,\hat{\boldsymbol{\imath}} + x^2 y\,\hat{\boldsymbol{\jmath}} + z\,\hat{\boldsymbol{k}}\,,$$

from point ***a*** to ***b*** using the path shown in the figure.



A3 [6 MARKS] Find the magnitude of the **electric field** and **the potential** inside a solid sphere of radius R and volume charge density of the form (considering the potential is zero at infinity)

$$\rho = \frac{k\,r}{R}.$$

A4 [6 MARKS] The electric potential in some spherical region is found to be

$$V = \frac{a}{r^2},$$

where a is an arbitrary constant. Find the **electric field** $\vec{E}$ and the **charge density** ρ.

PART [B]: SOLVE ONLY THREE QUESTIONS (B1 AND B2 ARE COMPULSORY)

B1 [6 MARKS] A one-dimensional conductor lies along the positive x-axis. The conductor is grounded (V = 0 V) at x = 2 m and at x = 10 m is kept at a constant potential of 160 V. Use Laplacian method to find the **potential** and **electric field** at x = 4 m.

B2 [6 MARKS] A sphere of radius R carries a polarization of the form

$$\vec{\boldsymbol{P}} = b\, r\, \hat{\boldsymbol{r}}$$

where b is a constant. **(i)** Calculate **the bound charges** ρ_b and σ_b. **(ii)** Find the electric field inside the sphere.

B3 [6 MARKS] Using the fact that the charge density ρ in a polarized material is the summation of the bound ρ_b and free ρ_f charges (i.e $\rho = \rho_b + \rho_f$), show that the electric displacement field $\vec{\boldsymbol{D}}$ is defined as

$$\vec{\boldsymbol{D}} = \varepsilon_0\, \vec{\boldsymbol{E}} + \vec{\boldsymbol{P}}$$

where $\vec{\boldsymbol{E}}$ is the electric field and $\vec{\boldsymbol{P}}$ is the polarization.

B3 [6 MARKS] A cable of a linear charge density λ is entirely surrounded by an insulating material of dielectric constant ε. Find **(i)** the susceptibility χ_e, **(ii)** the electric displacement field, and **(iii)** the polarization [Hint: use the results of **(i)** and **(ii)**].

WITH MY BEST WISHES