EE251
Lectures

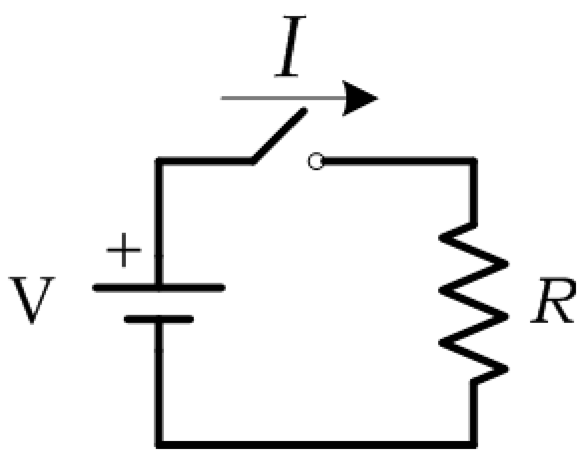
Circuit Analysis

Section 04

Ohm's Law

- A current through a resistor is proportional to the voltage across its terminals

$$V = I \cdot R \qquad or \qquad I = \frac{V}{R}$$



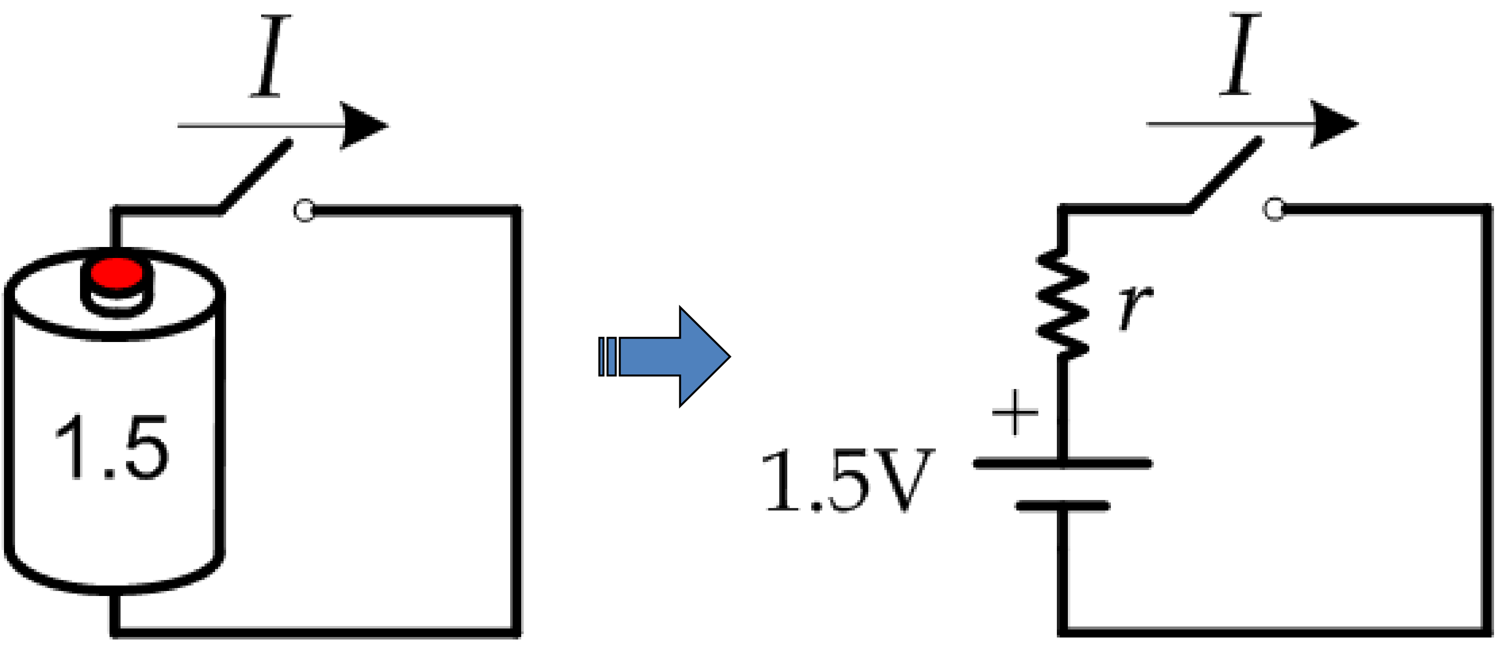
$$R \to \infty \quad I = 0$$

$$R \to 0 \quad I = \infty$$

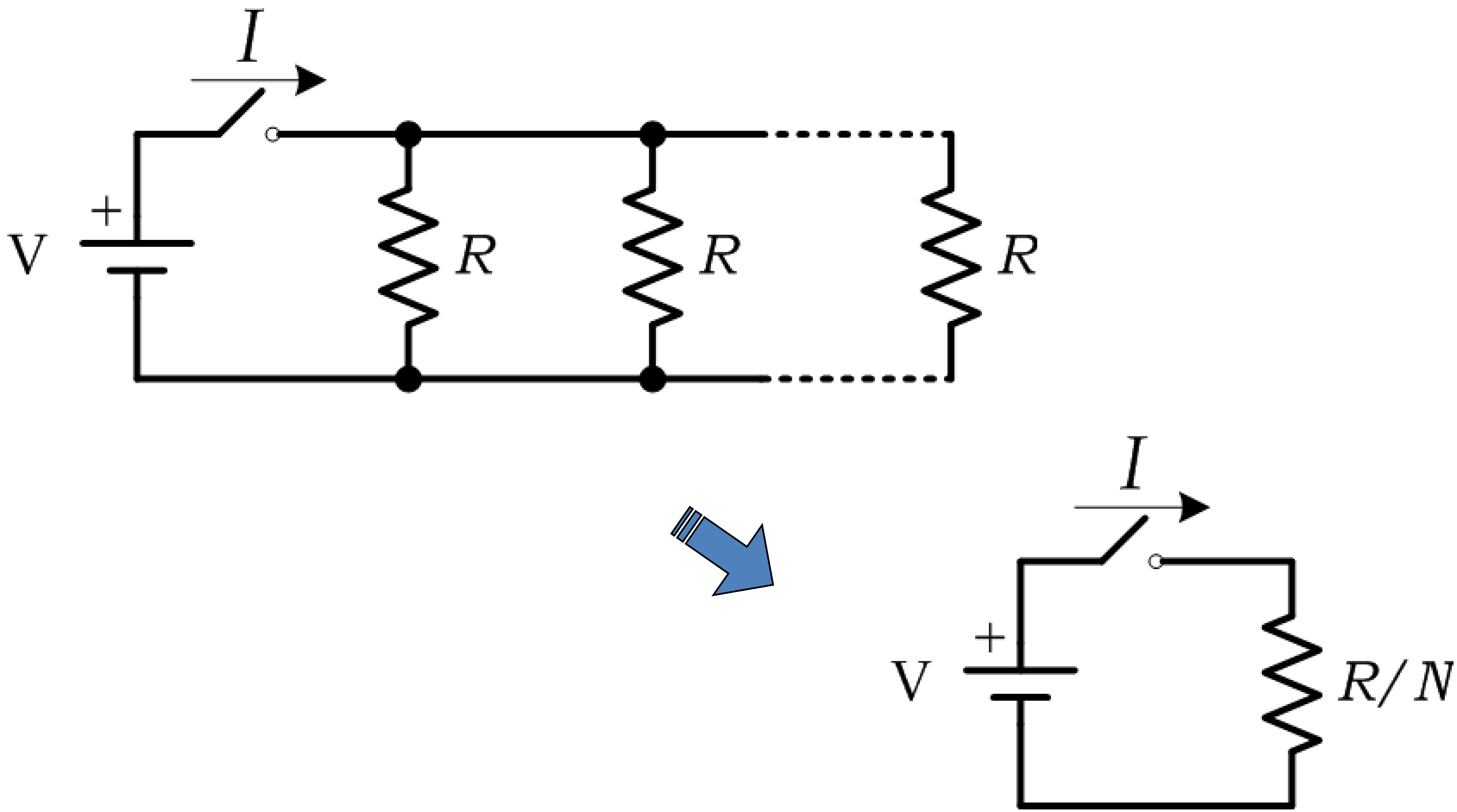
Short a 1.5 battery, what will happen?

Internal Resistance

- If you short a 1.5V battery, how much current will pass?

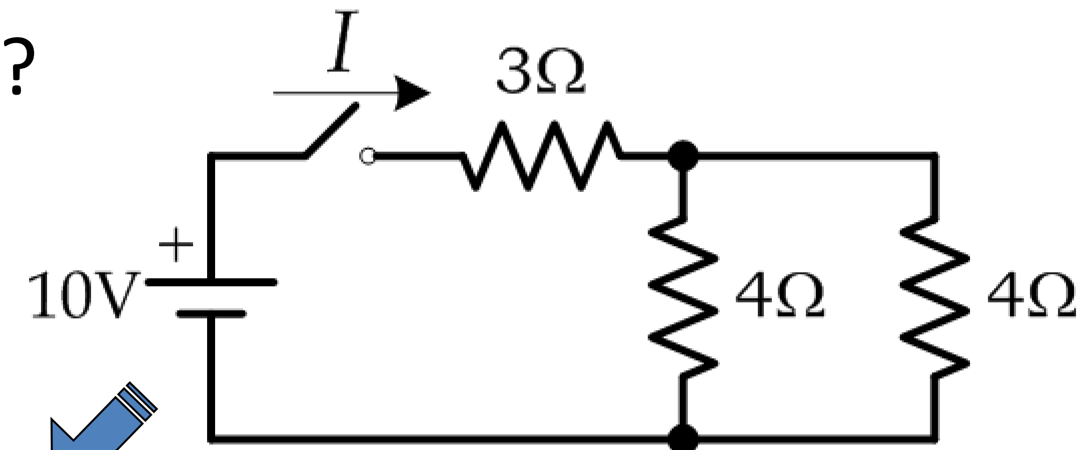


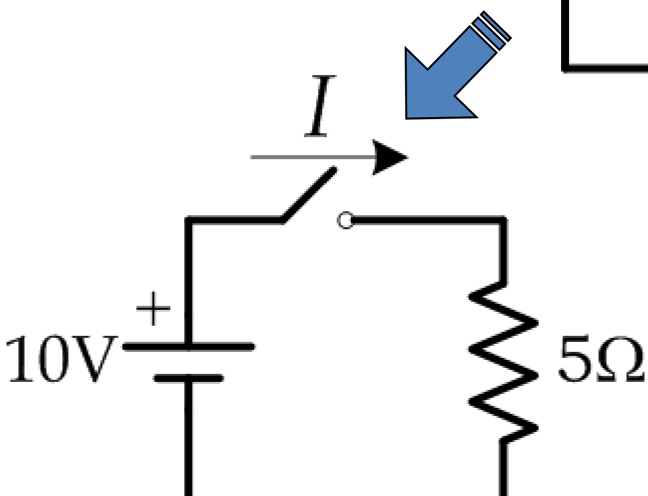
Overloading



Question

- What is the total current drawn from the source?

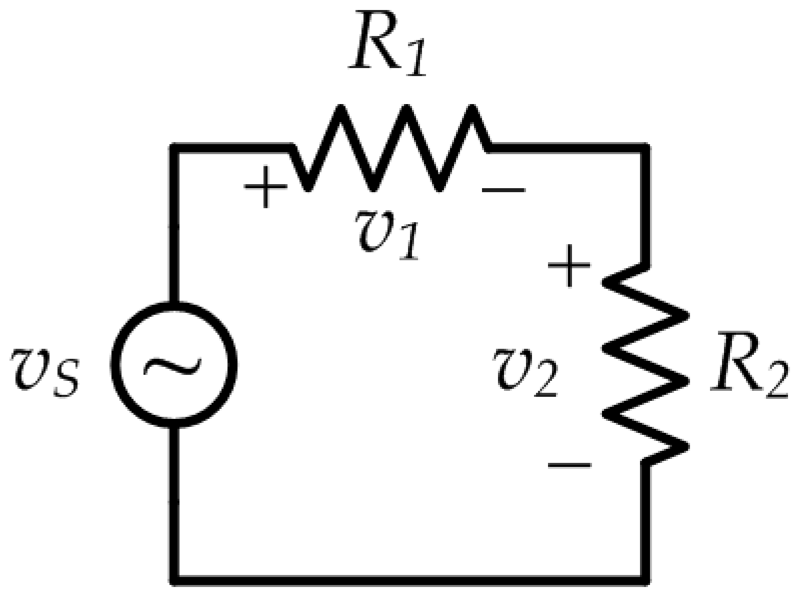




$$R_L = 4 // 4 + 3 = 2 + 3 = 5\Omega$$

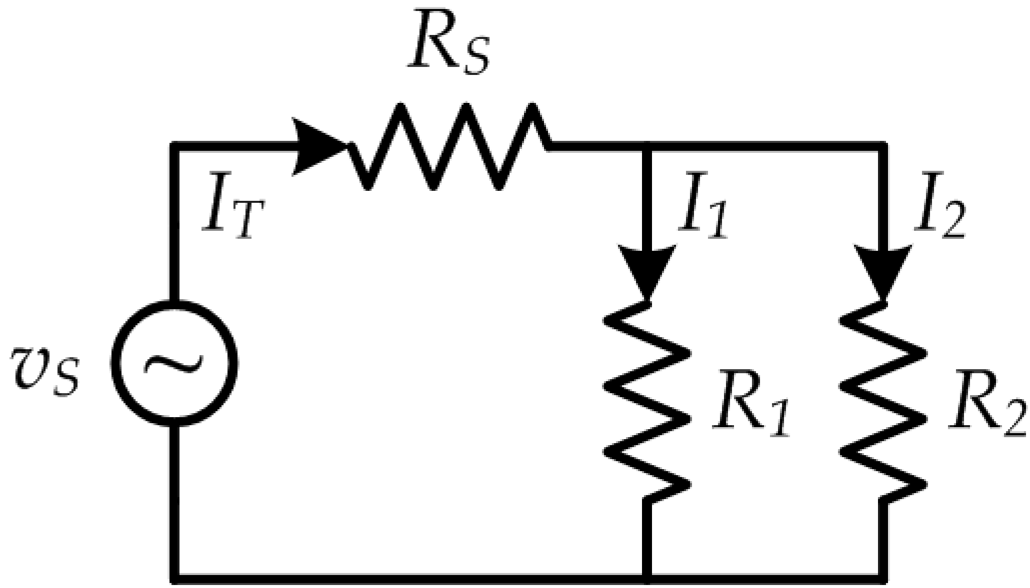
$$I = \frac{V}{R} = \frac{10}{5} = 2A$$

Voltage Divider



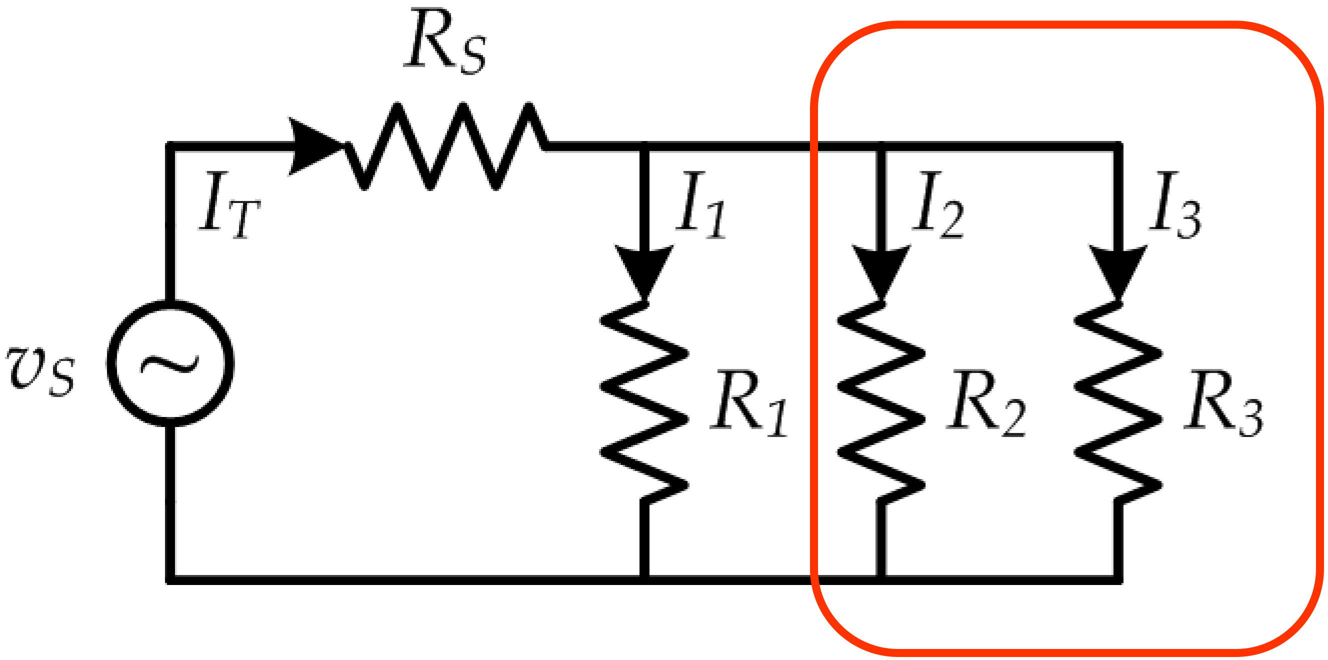
$$v_1 = v_s \frac{R_1}{R_1 + R_2} \qquad v_2 = v_s \frac{R_2}{R_1 + R_2}$$

Current Divider



$$I_1 = I_T \frac{R_2}{R_1 + R_2} \qquad I_2 = I_T \frac{R_1}{R_1 + R_2}$$

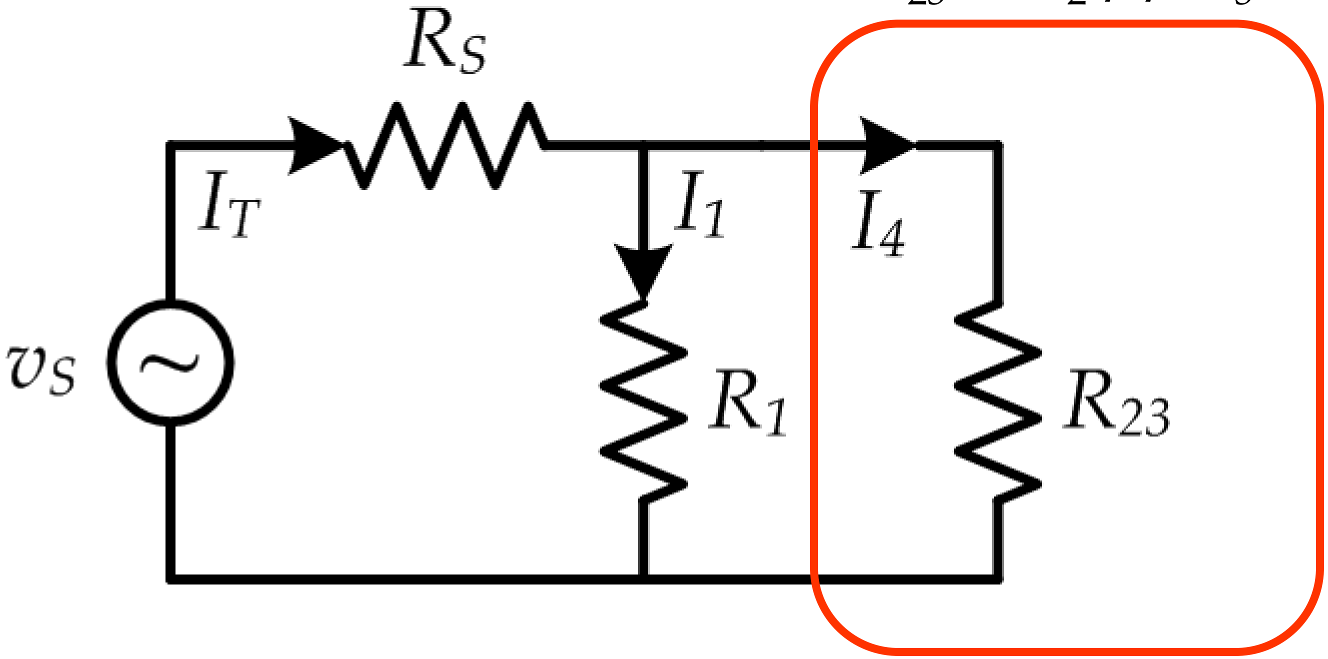
Multiple Loads



Combine Parallel Loads

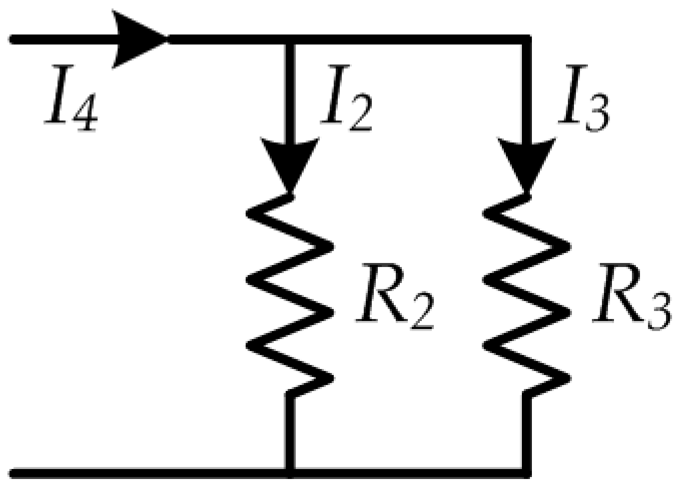
Multiple Loads

$$R_{23} = R_2 // R_3$$



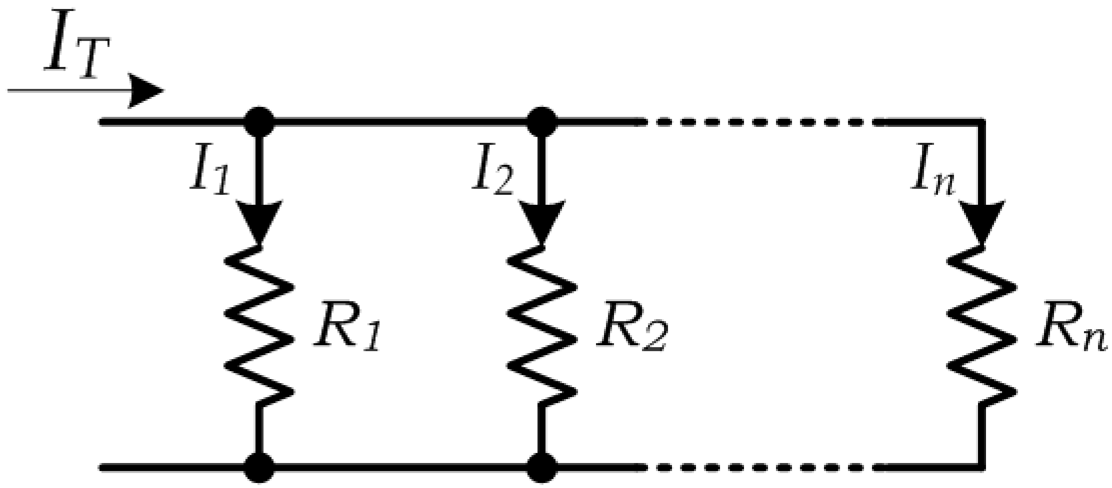
$$I_1 = I_T \frac{R_{23}}{R_1 + R_{23}} \qquad I_4 = I_T \frac{R_1}{R_1 + R_{23}}$$

Multiple Loads



$$I_2 = I_4 \frac{R_3}{R_2 + R_3} \qquad I_3 = I_4 \frac{R_2}{R_2 + R_3}$$

Current Divider

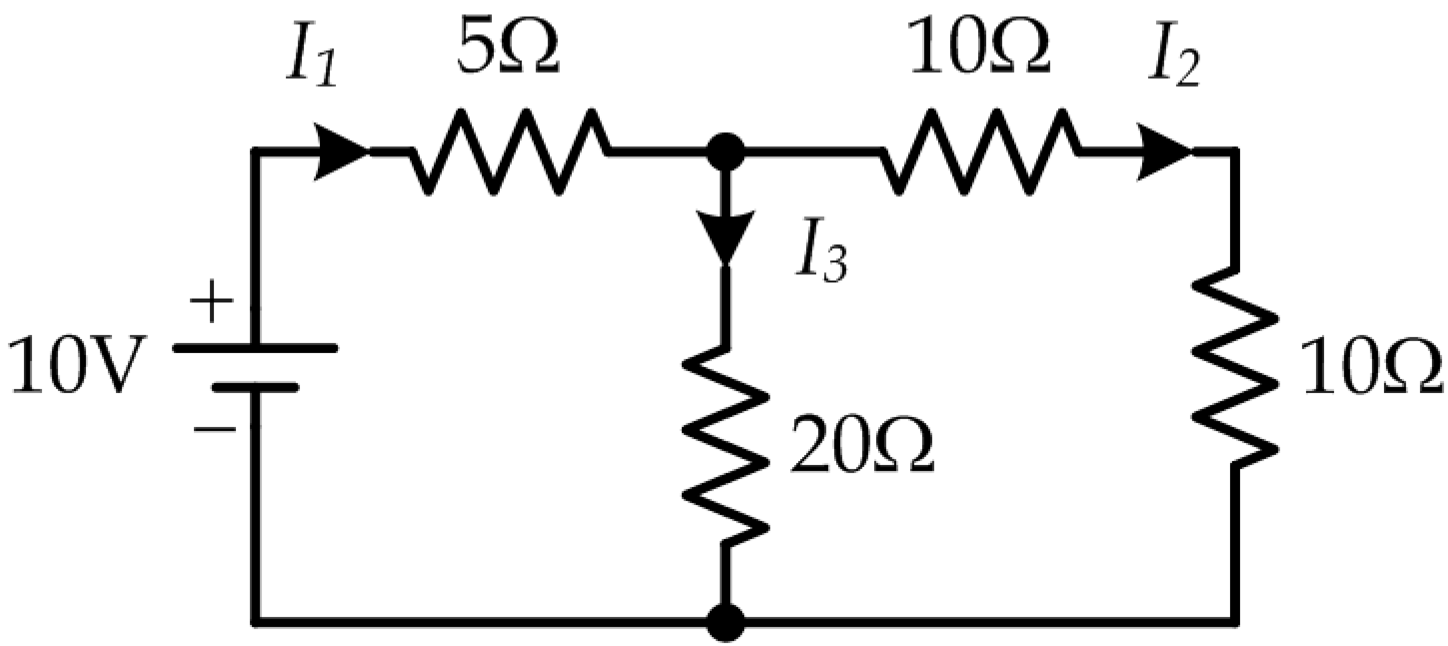


$$R_p = R_1 // R_2 // \cdots // R_n$$

$$I_k = I_T \frac{R_p}{R_k}$$

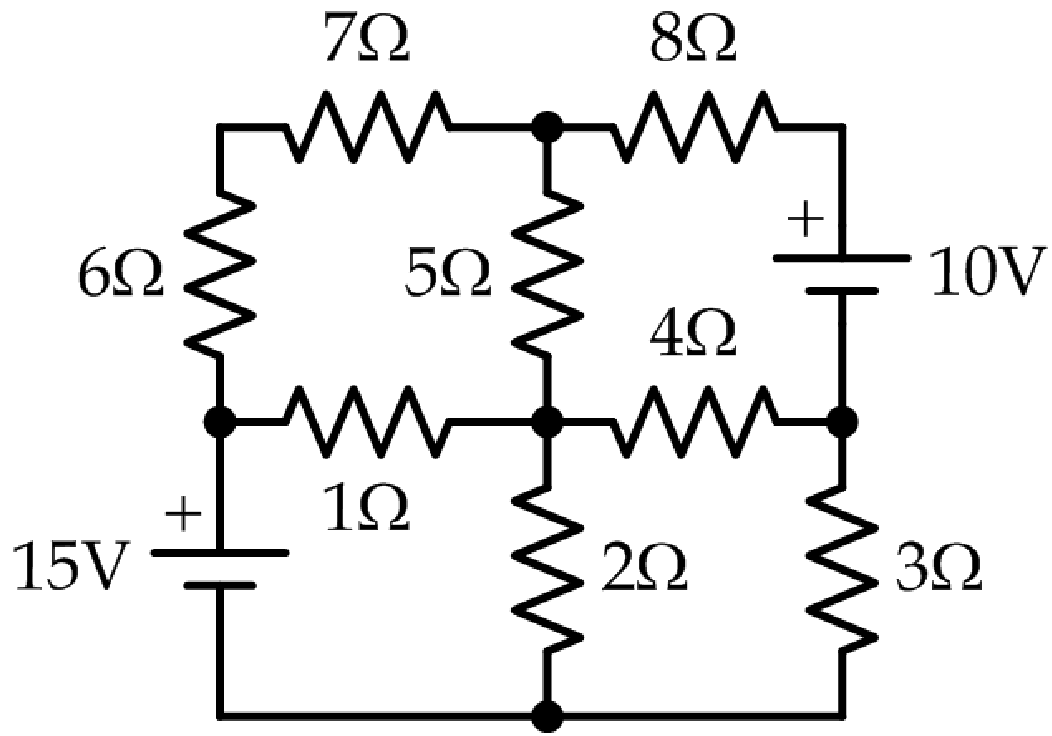
Process Check

- Calculate all currents in the circuit..
  - What is your plan?!



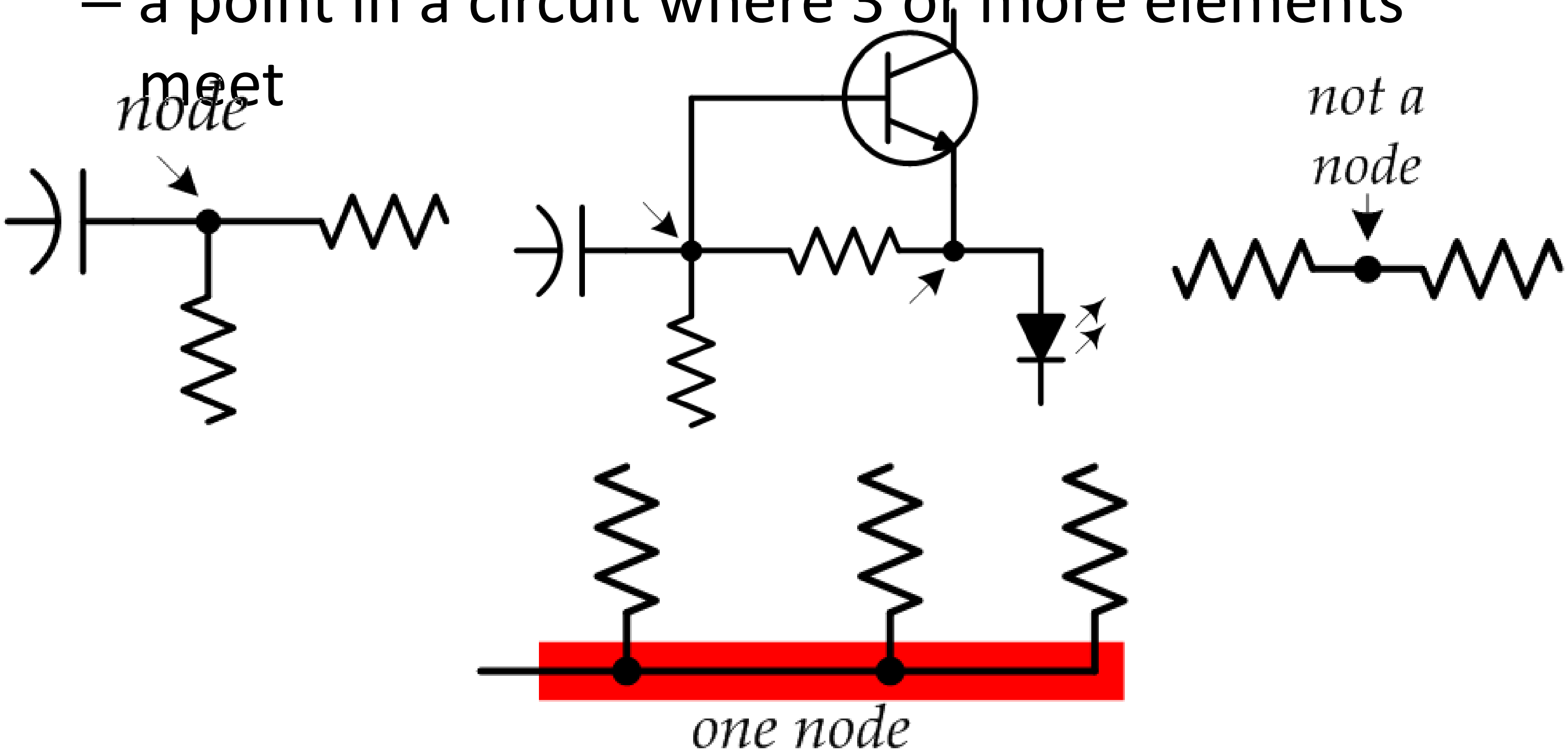
Multiple Sources

- What if more than one source in the circuit?
  - How to solve for all currents?



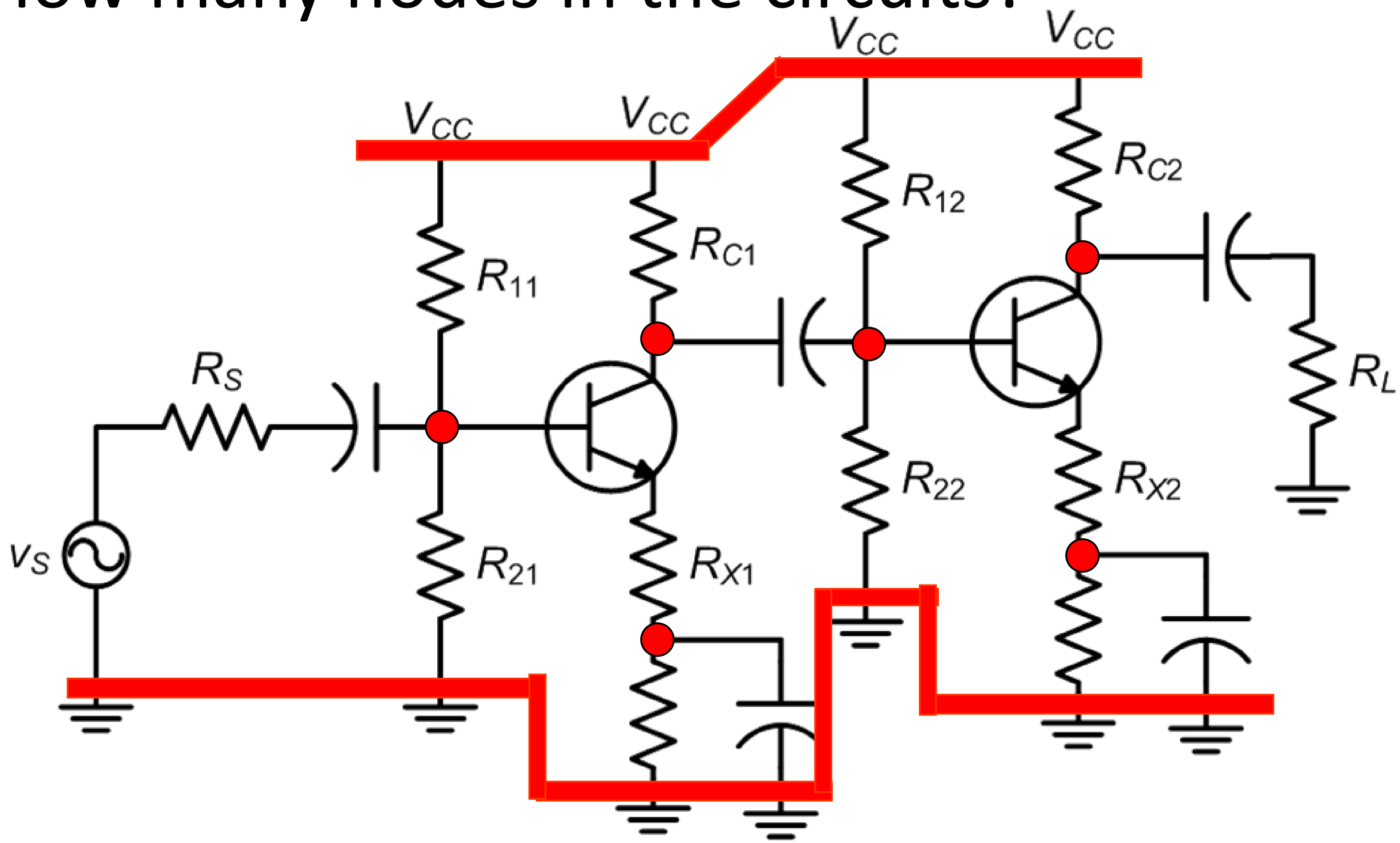
Nodes and Loops

- A Node
  - a point in a circuit where 3 or more elements meet



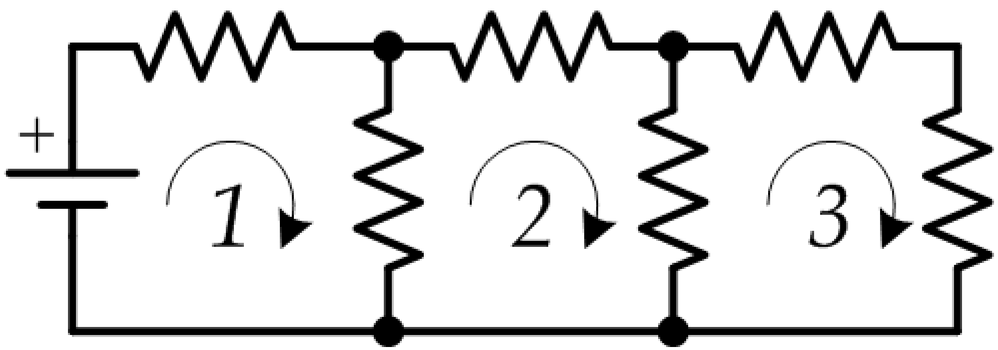
Nodes and Loops

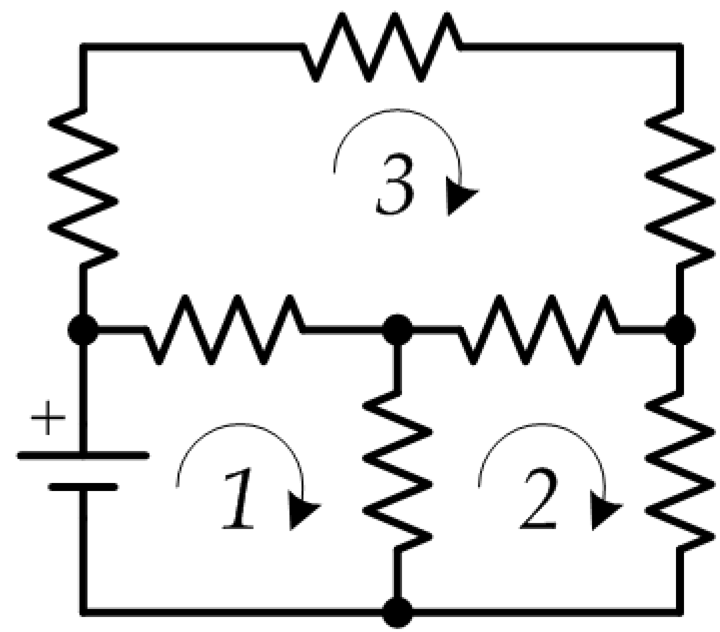
- How many nodes in the circuits?



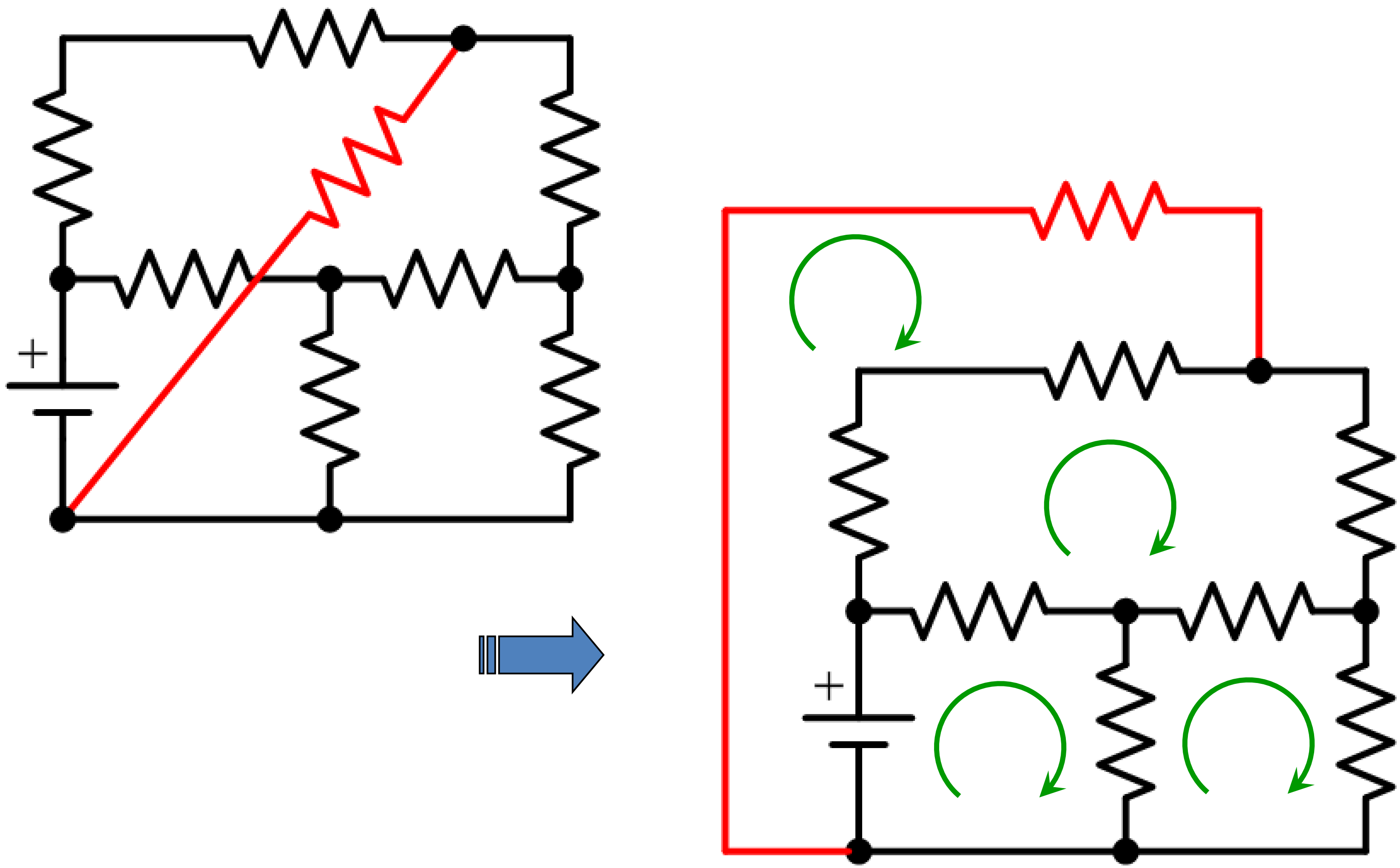
Nodes and Loops

- A Loop:
  - a closed ring in a planer circuit



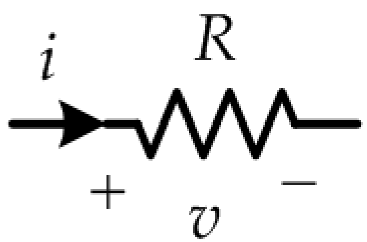


Planer Circuit

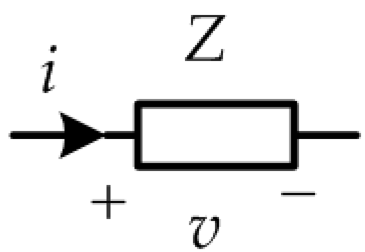


Voltage Drop

- A current passing through a load generates a voltage drop



$$v = i \cdot R$$



$$v = i \cdot Z$$

KVL and KCL

- *(nodes, loops, planner circuit, voltage drop) then what?*

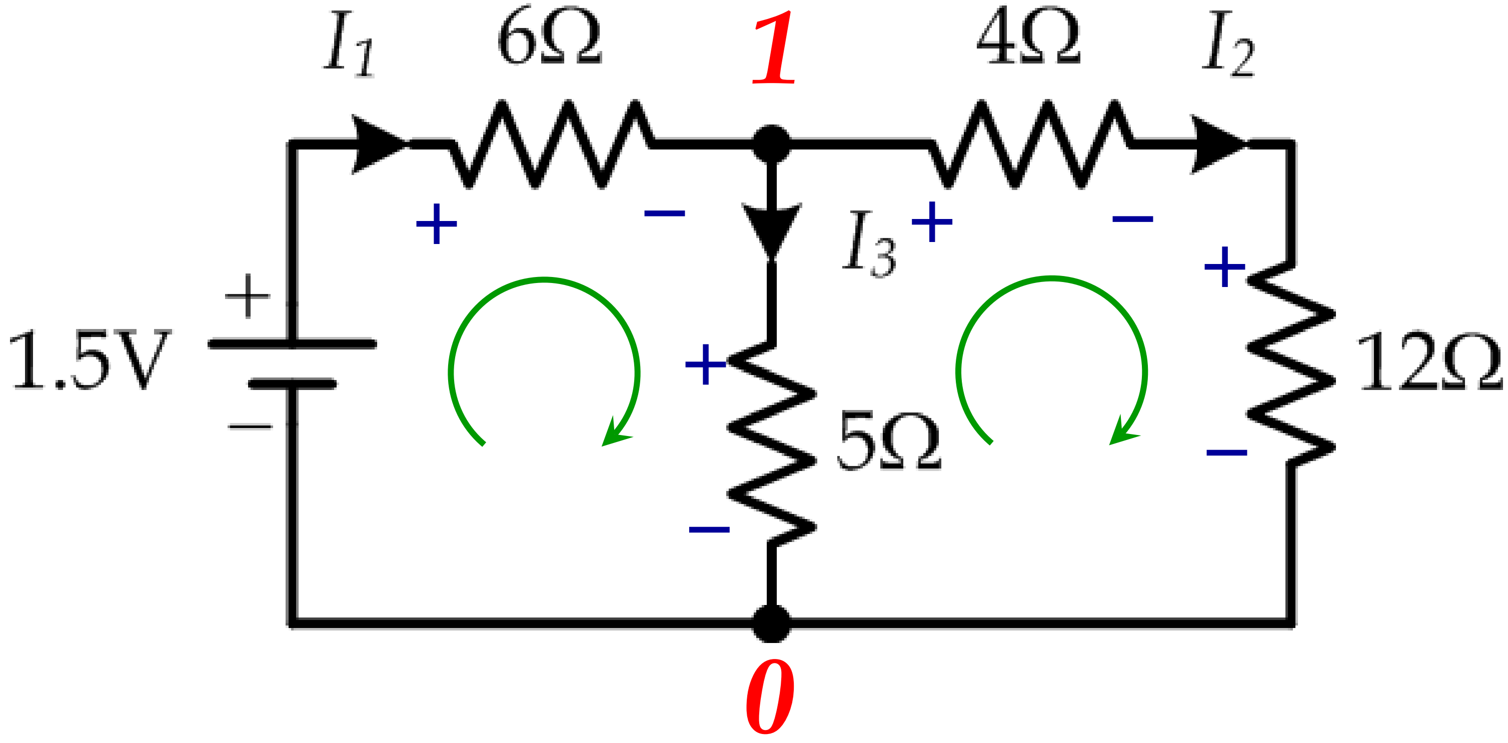
- To solve for all currents and volts in a circuit:
  - KVL: Kirchhoff's Voltage Law
    - the sum of voltage drops in a loop is zero
  - KCL: Kirchhoff's Current Law
    - the sum of currents into a node is zero

Circuit Analysis

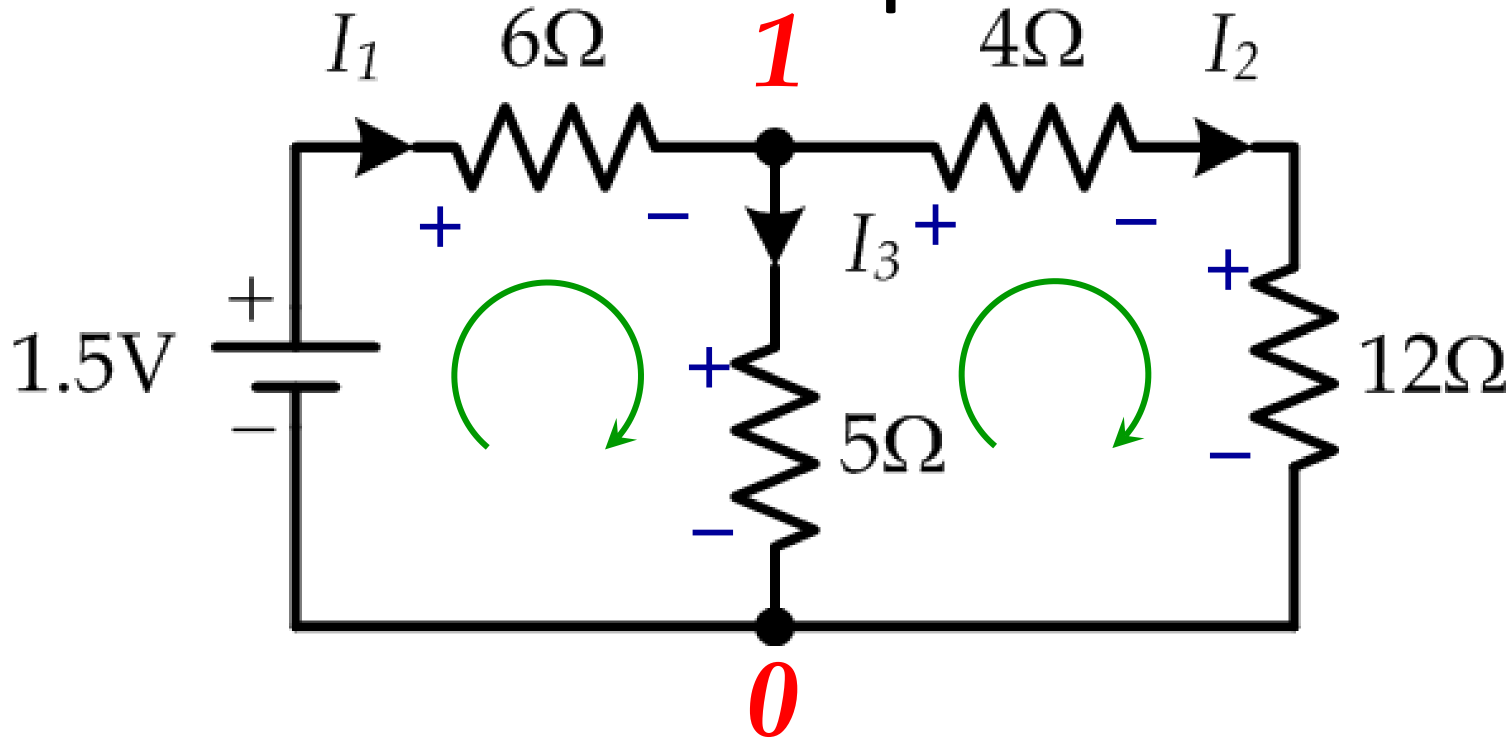
1. count nodes minus one (possible ground)
2. mark a current for each branch
3. write KCL equations

4. count loops
5. write KVL equations

Example



Example

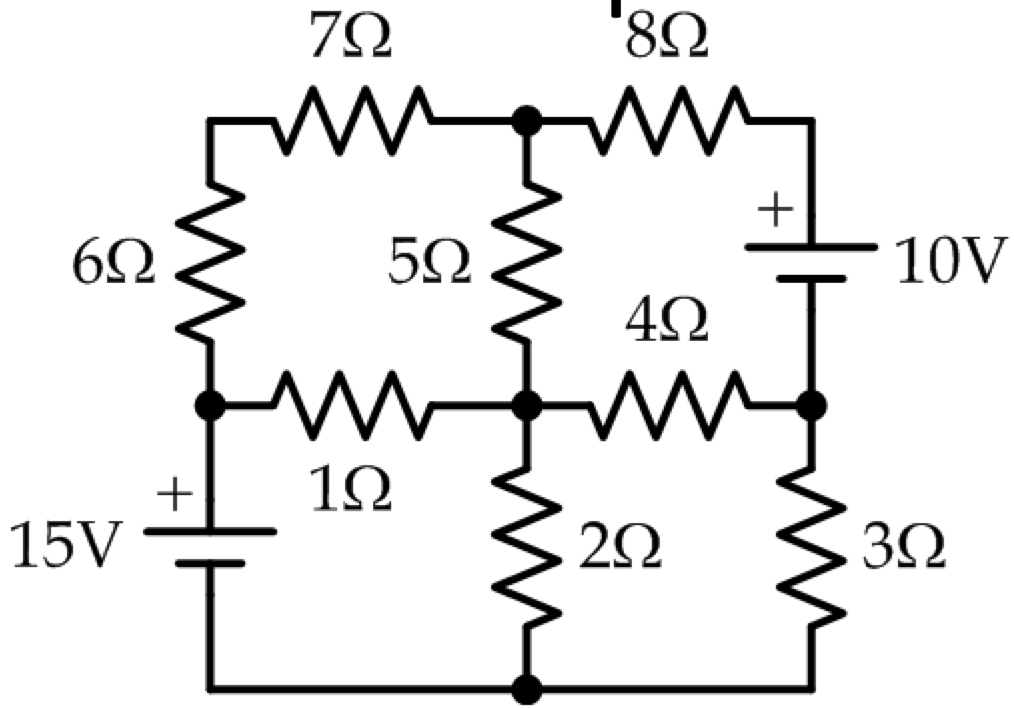


$$KCL: \quad I_1 = I_2 + I_3$$

$$KVL: \begin{aligned} -1.5 + 6I_1 + 5I_3 &= 0 \\ -5I_3 + 4I_2 + 12I_2 &= 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} I_1 &= 152.9 \quad mA \\ I_2 &= 36.4 \quad mA \\ I_3 &= 116.5 \quad mA \end{aligned}$$

Example



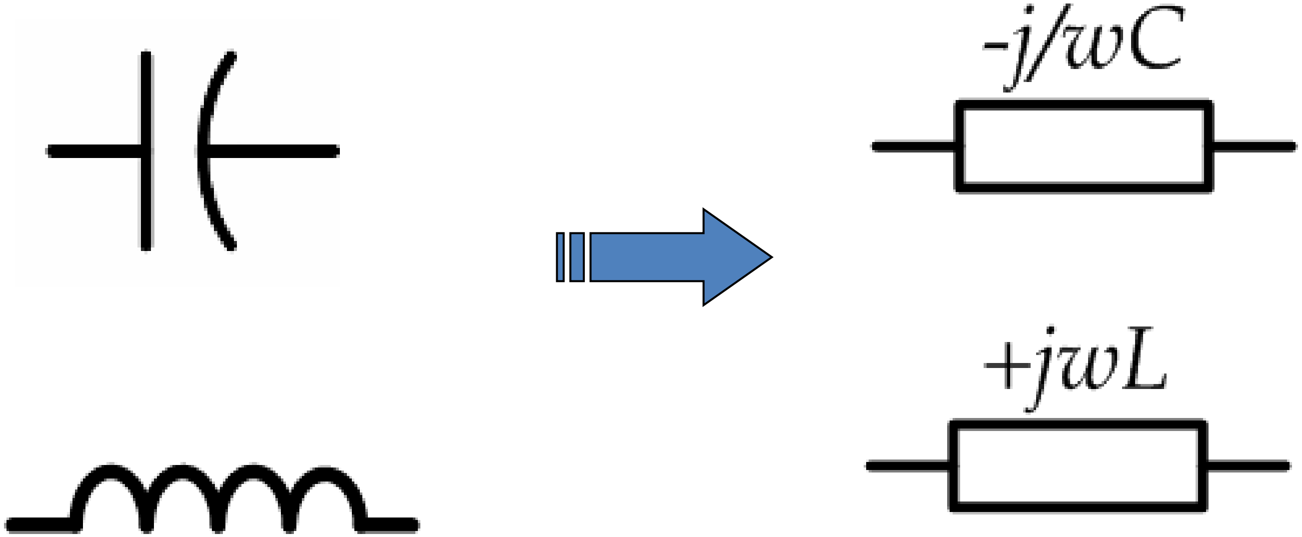
8 branches → 8 currents $I_1..I_8$ → 8 equations

4 nodes → 4 eqs

4 loops → 4 eqs

Impedance

- treat all passive components as resistors
  - but with *complex* resistances



Impedance

- What is the impedance of a 10μF capacitor when operated at 60Hz?

$$Z_C = -\frac{j}{wC} = -\frac{j}{2\pi \times 60 \times 10 \times 10^{-6}} = -j265.25\Omega$$

- What is the impedance of a 2mH inductor when operated at 60Hz?

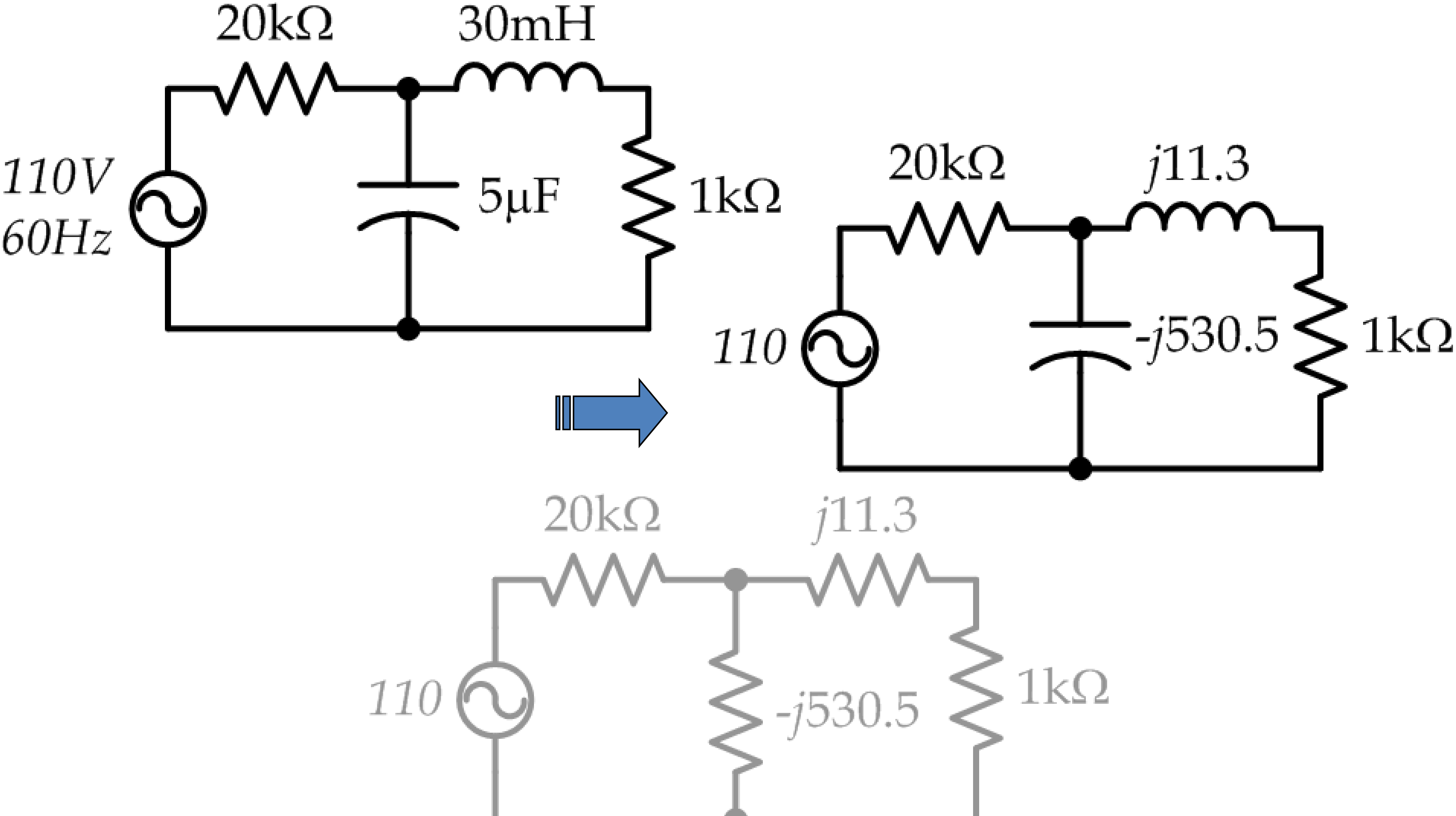
$$Z_L = jwL = j\,2\pi \times 60 \times 2 \times 10^{-3} = +j0.754\Omega$$

Laplace Domain

- When
  - Mixing AC and DC sources
  - Multiple different frequencies

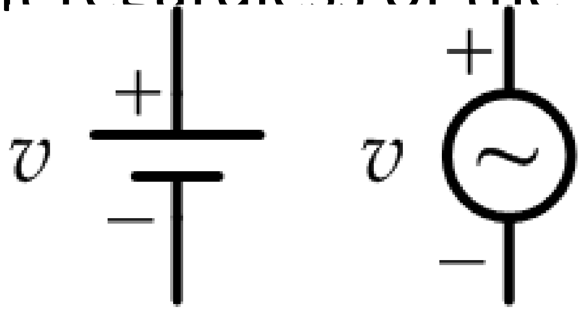
- use Laplace instead of Fourier
  - $jw \rightarrow s$
  - *Initial conditions*

Example

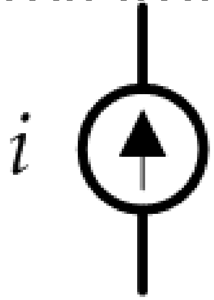

20kΩ
30mH
110V
60Hz
5μF
1kΩ
20kΩ
j11.3
110
-j530.5
1kΩ
20kΩ
j11.3
110
-j530.5
1kΩ

Current Source

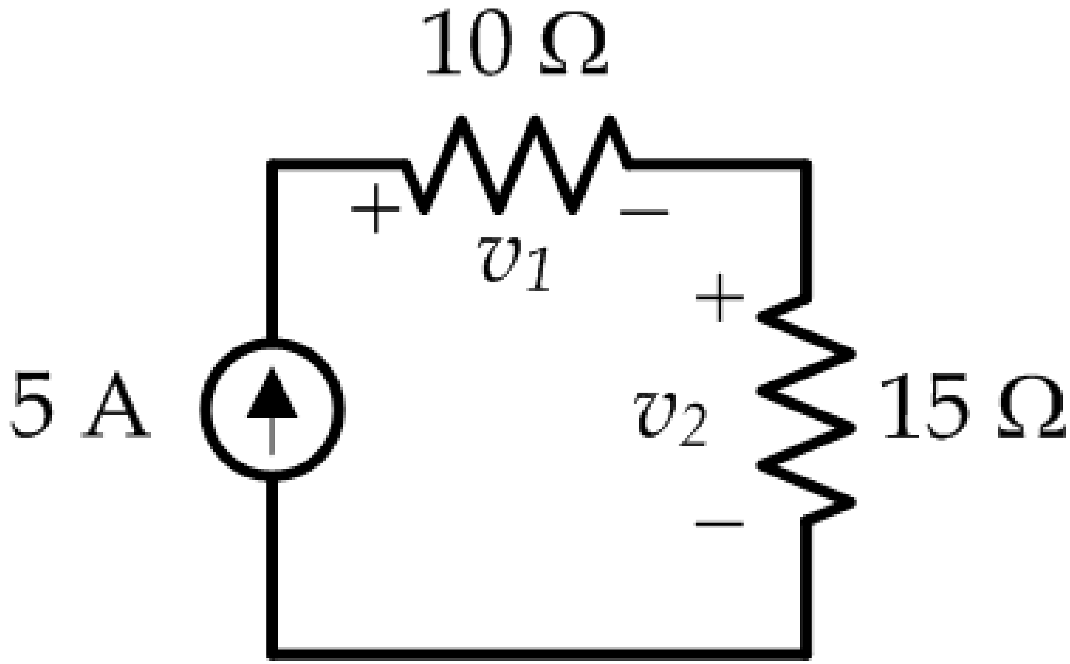
- Voltage Source
  - Generates constant volt regardless of the load



- Current Source
  - Generates constant current regardless of the load



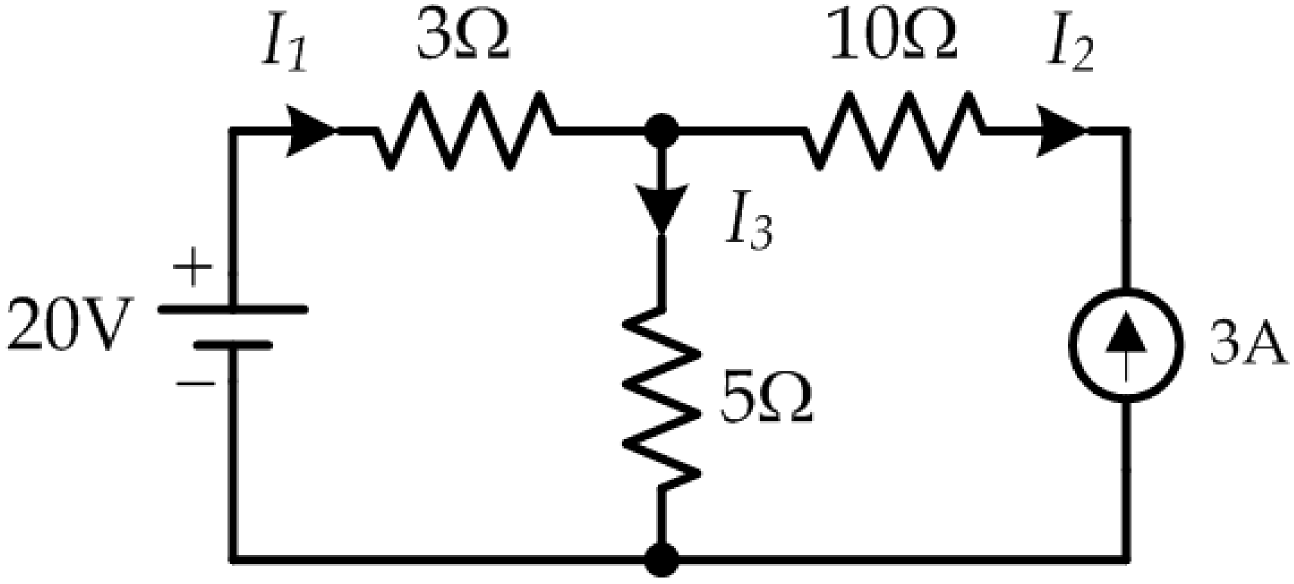
KVL & KCL



$$v_1 = 5 \times 10 = 50V$$

$$v_2 = 5 \times 15 = 75V$$

Example



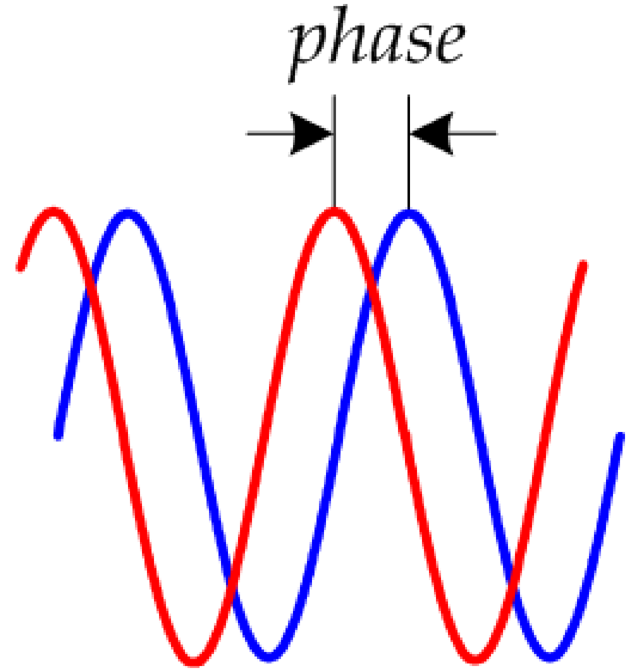
$$KCL: \quad I_1 = I_2 + I_3$$

$$KVL: \begin{aligned} -20 + 3I_1 + 5I_3 &= 0 \\ -5I_3 + 10I_2 + v_i &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} I_1 &= 0.625 \quad A \\ I_2 &= -3 \quad A \\ I_3 &= 3.625 \quad A \\ v_i &= 48.125 \quad V \end{aligned}$$

$$IS: \quad I_2 = -3A$$

Complex AC Source

- AC Volt or Current has:
  - Amplitude
  - Frequency
  - Phase



- Phase can be expressed in Complex Number

$$A\cos(2\pi f \cdot t + \varphi) \quad \rightarrow A\angle\varphi$$

Complex AC Source

$$A\cos\left(2\pi f\cdot t+\varphi\right)$$

$$A\angle\varphi \qquad \rightarrow A\cdot\left(\cos\varphi+j\sin\varphi\right)$$

$$110\angle 30^\circ \quad \rightarrow 110\cdot\left(\cos 30^\circ+j\sin 30^\circ\right)$$

$$=95.26+j55$$

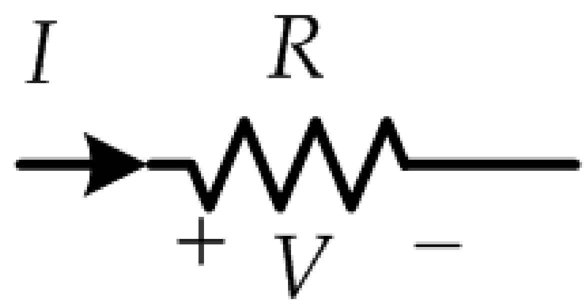
Power

- Power = rate of energy transfer
  - measured in Watts (W)

$$P = I \cdot V$$

$$P = I \cdot V = I^2 \cdot R$$

$$P = I \cdot V = \frac{V^2}{R}$$

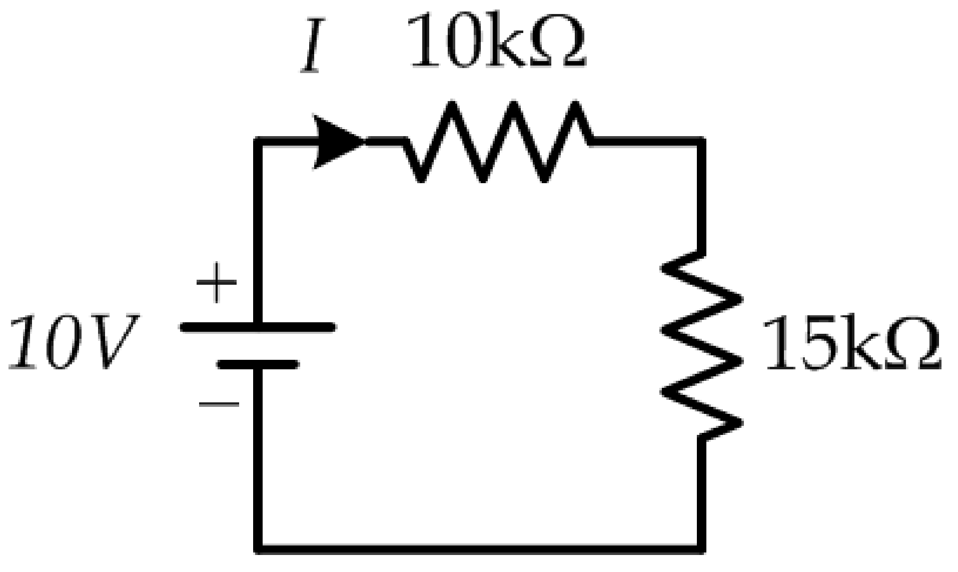


Reactive Power

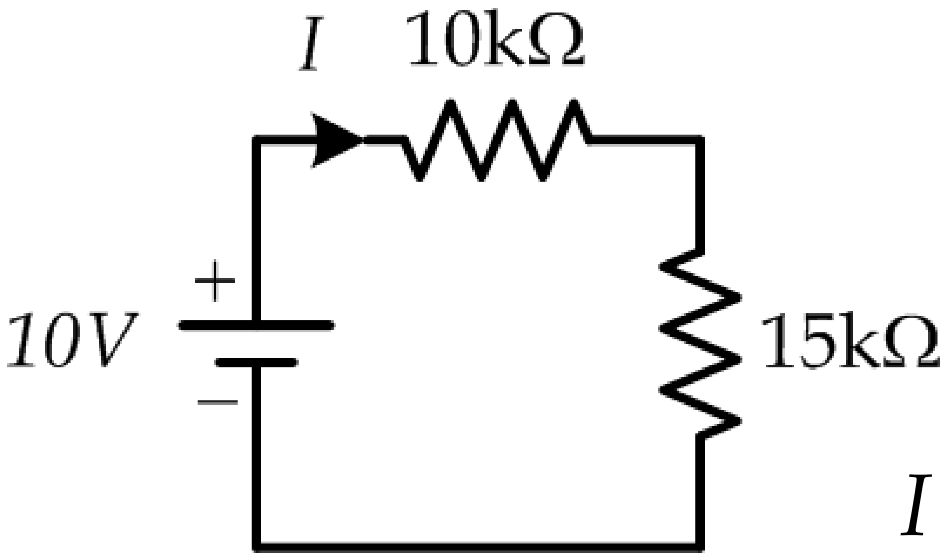
- Actual Power
  - On resistors only
    - $P = I \times V$
- Reactive Power
  - Imaginary Part of IV
    - $S = I^{*} \times V = P + jQ$
    - *RMS Values of (V) and (I)*

Process Check

- Solve the circuit shown and find the power consumption of each component



Solution (1)

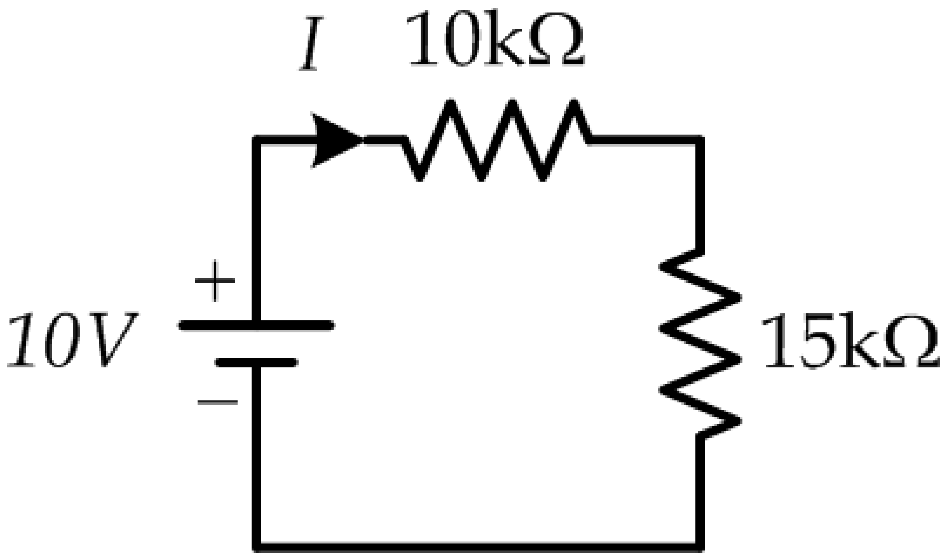


$$I = \frac{10}{10k + 15k} = 400\mu A$$

$$P_1 = I^2 \cdot R_1 = \left(400\times10^{-6}\right)^2 \cdot 10\times10^3 = 1.6mW$$

$$P_2 = I^2 \cdot R_2 = \left(400\times10^{-6}\right)^2 \cdot 15\times10^3 = 2.4mW$$

Solution (2)



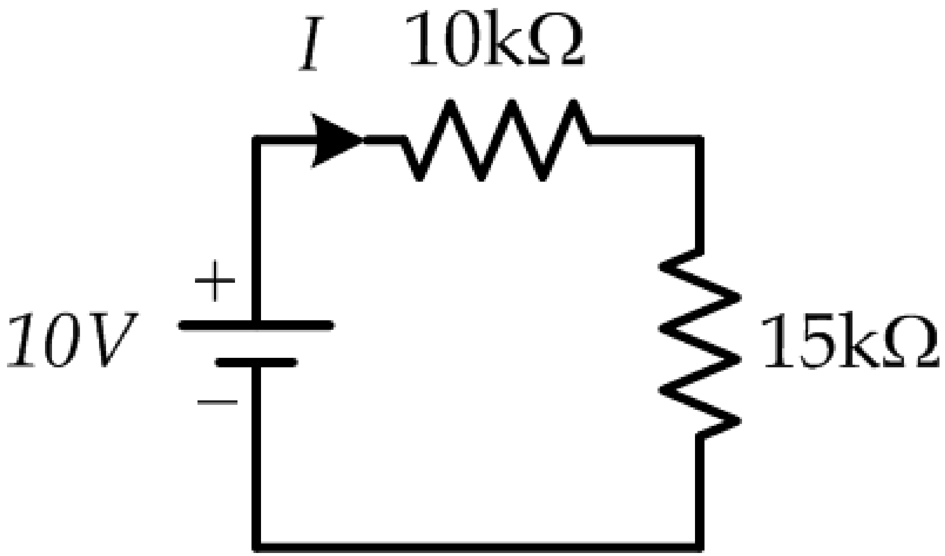
$$V_1 = 10\frac{10k}{10k+15k} = 4V$$

$$V_2 = 10\frac{15k}{10k+15k} = 6V$$

$$P_1 = \frac{V_1^2}{R_1} = \frac{4^2}{10\times10^3} = 1.6mW$$

$$P_2 = \frac{V_2^2}{R_2} = \frac{6^2}{15\times10^3} = 2.4mW$$

Total Power = 0



$$P_S = I \cdot V = -400 \times 10^{-6} \cdot 10 = -4mW$$

$$P_1 + P_2 = 4mW$$

Total Power = Zero