بسم الله الرحمن الرحيم

King Abdulaziz University

Review for sections 0.4 and 0.5

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على عشرون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus I
Math110

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Enter Name: [ ]

I.D. Number: [| | | | | | | | |]

Start Answer each of the following.

1. Covert the $75°$ to radian

- [ ] $5\pi/12$
- [ ] $\pi/12$
- [ ] $2\pi/12$
- [ ] $3\pi/12$

2. Covert the $\frac{4\pi}{9}$ to degree

- [ ] $40°$
- [ ] $160°$
- [ ] $80°$
- [ ] $240°$

3. $\sin\left(\frac{\pi}{3}\right) =$

- ☐ $\frac{1}{2}$
- ☐ $\frac{\sqrt{3}}{2}$
- ☐ $-\frac{1}{2}$
- ☐ $-\frac{\sqrt{3}}{2}$

4. $\cos\left(\frac{2\pi}{3}\right) =$

- ☐ $\frac{1}{2}$
- ☐ $\frac{\sqrt{3}}{2}$
- ☐ $-\frac{\sqrt{3}}{2}$
- ☐ $-\frac{1}{2}$

5. $\sin^2 x =$

- ☐ $\frac{1-\cos(2x)}{2}$
- ☐ $\frac{1+\cos(2x)}{2}$
- ☐ $\frac{1-\sin(2x)}{2}$
- ☐ $\frac{1+\sin(2x)}{2}$

6. $\cos^2 x =$

- ☐ $\frac{1-\cos(2x)}{2}$
- ☐ $\frac{1+\cos(2x)}{2}$
- ☐ $\frac{1-\sin(2x)}{2}$
- ☐ $\frac{1+\sin(2x)}{2}$

7. $\cos\left(x + \frac{\pi}{2}\right) =$

- ☐ $-\sin x$
- ☐ $\sin x$
- ☐ $-\cos x$
- ☐ $\cos x$

8. $\sin\left(x + \frac{\pi}{2}\right) =$

- ☐ $-\sin x$
- ☐ $\sin x$
- ☐ $-\cos x$
- ☐ $\cos x$

9. $\cos^2 x + \sin^2 x =$.

- ☐ 0
- ☐ 1
- ☐ -1

10. $2\sin x \cos x =$.

- ☐ $\cos(2x)$
- ☐ 1
- ☐ $\sin^2 x$
- ☐ $\sin(2x)$

11. If $f(x) = x^2 - x + 1$ and $g(x) = x - 1$, then $(fg)(x) = .$

- ☐ $x^3 - 2x^2 + 2x + 1$
- ☐ $x^3 - 2x^2 + 2x - 1$
- ☐ $x^3 + 2x^2 + 2x - 1$
- ☐ $x^3 - 2x^2 - 2x - 1$

12. If $f(x) = \sqrt{x-3}$ and $g(x) = x + 1$, $\left(\frac{g}{f}\right)(x) =$

- ☐ $\frac{\sqrt{x-3}}{x+1}$
- ☐ $\frac{x+1}{\sqrt{x-3}}$
- ☐ $\sqrt{\frac{x-3}{x+1}}$
- ☐ $\frac{x-3}{x+1}$

13. If $f(x) = \sqrt{x-3}$ and $g(x) = x+1$, then domain $\left(\frac{g}{f}\right)$ is

- ☐ $\mathbb{R}$
- ☐ $(-\infty, 3)$
- ☐ $(3, \infty)$
- ☐ $[3, \infty)$

14. Find the domain of the function $f(x) = \frac{\sqrt[6]{9-x^2}}{\sqrt[3]{x-2}}$.

- ☐ $\mathbb{R}$
- ☐ $[-3, 2) \cup (2, 3]$
- ☐ $\mathbb{R} \setminus \{2\}$
- ☐ $(-\infty, -3]$

15. Find the domain of the function $f(x) = \sqrt{x-1}\sqrt[3]{-5x-6}$.

- ☐ $[-6/5, \infty)$
- ☐ $\mathbb{R} \setminus \{-6/5, 1\}$
- ☐ $(-\infty, -6/5] \cup [1, \infty)$
- ☐ $[1, \infty)$

16. Find the domain of the function

$$f(x) = \sqrt[4]{2x+8} - \frac{5x+1}{x^2-x-6}.$$

- ☐ $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$
- ☐ $[-4, -2) \cup (-2, 3) \cup (3, \infty)$
- ☐ $(-4, -2) \cup (-2, 3) \cup (3, \infty)$
- ☐ $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

17. If $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + 1$, $(f \circ g)(x) =$.

- ☐ $\sqrt{x^2 - 1}$
- ☐ $x - 1$
- ☐ $\sqrt{x^2 + 1}$
- ☐ $\sqrt{x-2} + 1$

18. If $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + 1$, then domain $f \circ g$ is

- ☐ $\mathbb{R}$
- ☐ $\mathbb{R} \setminus \{1, -1\}$
- ☐ $(-\infty, -1] \cup [1, \infty)$
- ☐ $[1, \infty)$

19. Let $g(x) = \dfrac{1}{x^2 - 1}$ and $f(x) = \sqrt{x+1}$, then $(g \circ f)(x) =$

- ☐ $\frac{1}{x}$
- ☐ $\frac{1}{x+2}$
- ☐ $\frac{1}{\sqrt{x+1}-1}$
- ☐ $\sqrt{\frac{1}{x^2-1} + 1}$

20. Let $g(x) = \dfrac{1}{x^2 - 1}$ and $f(x) = \sqrt{x+1}$, then $D(g \circ f) =$

- ☐ $(-\infty, 0) \cup (0, \infty)$
- ☐ $\mathbb{R}$
- ☐ $[-1, \infty)$
- ☐ $[-1, 0) \cup (0, \infty)$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$75^\circ = \cancel{75}^{5} \cdot \frac{\pi}{\cancel{180}^{12}}$$
$$= \frac{5\pi}{12}$$

■

Solution to 2.

$$\frac{4\pi}{9} = \frac{4\cancel{\pi}}{\cancel{9}} \cdot \frac{\cancel{180}^{20}}{\cancel{\pi}}$$
$$= 4 \cdot 20 = 80^\circ$$

■

Solution to 3. We see from the table below that

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

■

Solution to 4.

$$\begin{aligned}\cos\left(\frac{2\pi}{3}\right) &= \cos\left(\pi - \frac{\pi}{3}\right)\\ &= \cos(\pi)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)\sin(\pi)\\ &= (-1)(\frac{1}{2}) - (\frac{\sqrt{3}}{2})(0)\\ &= -\frac{1}{2}\end{aligned}$$

■

Solution to 5.

$$\begin{aligned}
\cos(2x) &= \cos(x+x) && \text{use } \cos(x+y) = \cos x\cos y - \sin y\sin x \\
&= \cos x\cos x - \sin x\sin x && \cos x\cdot\cos x = \cos^2 x \text{ and } \sin x\cdot\sin x = \sin^2 x, \\
&= \cos^2 x - \sin^2 x && \cos^2 x + \sin^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \sin^2 x \\
\cos(2x) &= 1 - \sin^2 x - \sin^2 x && \text{add and solve for } \sin^2 x \\
\cos(2x) &= 1 - 2\sin^2 x && \text{move } -2\sin^2 x \text{ to the other side} \\
2\sin^2 x &= 1 - \cos(2x) && \text{divide by 2} \\
\sin^2 x &= \frac{1-\cos(2x)}{2}
\end{aligned}$$

■

Solution to 6.

$$\begin{aligned}
\cos(2x) &= \cos(x+x) && \text{use } \cos(x+y) = \cos x\cos y - \sin y\sin x\\
&= \cos x\cos x - \sin x\sin x && \cos x\cdot\cos x = \cos^2 x \text{ and } \sin x\cdot\sin x = \sin^2 x,\\
&= \cos^2 x - \sin^2 x && \cos^2 x + \sin^2 x = 1 \Leftrightarrow \sin^2 x = 1 - \cos^2 x\\
\cos(2x) &= \cos^2 x - (1 - \cos^2 x) &&\\
\cos(2x) &= \cos^2 x - 1 + \cos^2 x && \text{add and solve for } \cos^2 x\\
\cos(2x) &= 2\cos^2 x - 1 && \text{add and solve for } \cos^2 x\\
2\cos^2 x &= 1 + \cos(2x) && \text{divide by 2}\\
\cos^2 x &= \frac{1+\cos(2x)}{2} &&
\end{aligned}$$

■

Solution to 7.

$$\cos\left(x+\frac{\pi}{2}\right) = \cos x\cos\left(\frac{\pi}{2}\right) - \sin x\sin\left(\frac{\pi}{2}\right) \qquad \cos(x+y) = \cos x\cos y - \sin y\sin x,$$

$$= \cos x\cdot 0 - \sin x\cdot 1 \qquad \cos\left(\frac{\pi}{2}\right) = 0, \sin\left(\frac{\pi}{2}\right) = 1$$

$$= 0 - \sin x$$

$$= -\sin x$$

■

Solution to 8.

$$\begin{aligned}\sin\left(x+\frac{\pi}{2}\right) &= \sin x\cos\left(\frac{\pi}{2}\right)+\cos x\sin\left(\frac{\pi}{2}\right) && \sin(x+y)=\sin x\cos y+\sin y\cos x,\\ &= \sin x\cdot 0+\cos x\cdot 1 && \cos\left(\frac{\pi}{2}\right)=0, \sin\left(\frac{\pi}{2}\right)=1\\ &= 0+\cos x\\ &= \cos x\end{aligned}$$

■

Solution to 9. $\cos^2 x + \sin^2 x = 1.$ ■

Solution to 10.

$$\begin{aligned} \sin(2x) &= \sin(x+x) && 2x = x+x \\ &= \sin x\cos x + \sin x\cos x && \sin(x \pm y) = \sin x\cos y \pm \sin y\cos x, \\ &= 2\sin x\cos x \end{aligned}$$

■

Solution to 11.

$$\begin{aligned}(fg)(x) &= f(x)\cdot g(x) && \text{by definition.}\\ &= (x^2-x+1)(x-1) && \text{use multiplication}\\ &= (x^2-x+1)x-(x^2-x+1)\\ &= x^3-x^2+x-x^2+x-1 = x^3-2x^2+2x-1.\end{aligned}$$

■

Solution to 12. $\left(\dfrac{g}{f}\right)(x) = \dfrac{g(x)}{f(x)} = \dfrac{x+1}{\sqrt{x-3}}.$ ■

Solution to 13. The function $f(x) = \sqrt{x-3}$ is an even root function, hence f is defined if

$$x - 3 \geq 0 \Leftrightarrow x \geq 3.$$

Hence $D(f) = [3, \infty)$. The function $g(x) = x + 1$, is a polynomial, hence $D(g) = \mathbb{R}$. Now,

$$\begin{aligned} D(g/f) &= (D(g) \cap D(f)) \setminus \{x : f(x) = 0\} \\ &= (\mathbb{R} \cap [3, \infty)) \setminus \{x : \sqrt{x-3} = 0\} \\ &= [3, \infty) \setminus \{x : x - 3 = 0\} = [3, \infty) \setminus \{3\} \\ &= (3, \infty). \end{aligned}$$

■

Solution to 14. The function $f(x) = \dfrac{\sqrt[6]{9-x^2}}{\sqrt[3]{x-2}}$ is the quotient of $\sqrt[6]{9-x^2}$ and $\sqrt[3]{x-2}$.

Hence $D(f) = D(\sqrt[6]{9-x^2}) \cap D(\sqrt[3]{x-2}) \setminus \{x : \sqrt[3]{x-2} = 0\}$.

The function $\sqrt[6]{9-x^2}$ is an even root function, then

$$\begin{aligned}
9 - x^2 \geq 0 &\Leftrightarrow x^2 \geq 9 && \text{move } x^2 \text{ to the other side} \\
&\Leftrightarrow \sqrt{x^2} \leq 3 && \text{take the square root} \\
&\Leftrightarrow |x| \leq 3 && \sqrt{x^2} = |x| \text{ use properties of} \\
&\Leftrightarrow -3 \leq x \leq 3 && \text{absolute value inequality}
\end{aligned}$$

Hence $D(\sqrt[6]{9-x^2}) = [-3, 3]$.

The function $\sqrt[3]{x-2}$ is an odd root function, hence $D(\sqrt[3]{x-2}) = \mathbb{R}$.

$$\begin{aligned}
D(f) &= D(\sqrt[6]{9-x^2}) \cap D(\sqrt[3]{x-2}) \setminus \{x : \sqrt[3]{x-2} = 0\} \\
&= [-3, 3] \cap \mathbb{R} \setminus \{x : x-2 = 0\} \\
&= [-3, 3] \setminus \{2\} = [-3, 2) \cup (2, 3].
\end{aligned}$$

■

Solution to 15. The function $f(x) = \sqrt{x-1}\sqrt[3]{-5x-6}$ is the product of $\sqrt{x-1}$ and $\sqrt[3]{-5x-6}$.
Hence $D(f) = D(\sqrt{x-1}) \cap D(\sqrt[3]{-5x-6})$. The function $\sqrt{x-1}$ is an even root function, hence $x - 1 \geq 0 \Leftrightarrow x \geq 1$.
Thus $D(\sqrt{x-1}) = [1, \infty)$.
The function $\sqrt[3]{-5x-6}$ is an odd function and hence $D(\sqrt[3]{-5x-6}) = \mathbb{R}$.

$$\begin{aligned} D(f) &= D(\sqrt{x-1}) \cap D(\sqrt[3]{-5x-6}) \\ &= [1, \infty) \cap \mathbb{R} \\ &= [1, \infty). \end{aligned}$$

■

Solution to 16. The function $f(x) = \sqrt[4]{2x+8} - \frac{5x+1}{x^2-x-6}$ is the difference of $\sqrt[4]{2x+8}$ and $\frac{5x+1}{x^2-x-6}$.
Hence $D(f) = D(\sqrt[4]{2x+8}) \cap D(\frac{5x+1}{x^2-x-6})$.
The function $\sqrt[4]{2x+8}$ is an even root function, hence
$2x+8 \geq 0 \Leftrightarrow 2x \geq -8 \Leftrightarrow x \geq -4$.
Thus $D(\sqrt[4]{2x+8}) = [-4, \infty)$.
The function $\frac{5x+1}{x^2-x-6}$ is a rational function and hence

$$\begin{aligned} D(\frac{5x+1}{x^2-x-6}) &= \mathbb{R} \setminus \{x : x^2 - x - 6 = 0\} \\ &= \mathbb{R} \setminus \{-2, 3\} \\ &= (-\infty, -2) \cup (-2, 3) \cup (3, \infty). \end{aligned}$$

$$\begin{aligned} D(f) &= D(\sqrt[4]{2x+8}) \cap D(\frac{5x+1}{x^2-x-6}) \\ &= [-4, \infty) \cap \{(-\infty, -2) \cup (-2, 3) \cup (3, \infty)\} \\ &= [-4, -2) \cup (-2, 3) \cup (3, \infty). \end{aligned}$$

■

Solution to 17.

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f(x^2 + 1) \\
&= \sqrt{x^2 + 1 - 2} = \sqrt{x^2 - 1}.
\end{aligned}
$$

■

Solution to 18. $D(f \circ g) = D(g) \cap D(f(g(x)))$. Since $(f \circ g)(x) = f(g(x)) = \sqrt{x^2 - 1}$ then

$$\begin{aligned} x^2 - 1 \geq 0 &\Leftrightarrow \sqrt{x^2} \geq 1 && \text{take the square root} \\ &\Leftrightarrow |x| \geq 1 && \sqrt{x^2} = |x| \text{ use properties of} \\ &\Leftrightarrow x \geq 1 \text{ or } x \leq -1 && \text{absolute value inequality} \end{aligned}$$

Hence $D(f(g(x))) = (-\infty, -1] \cup [1, \infty)$. Since $g(x) = x^2 + 1$, is a polynomial, then $D(g) = \mathbb{R}$.

$$\begin{aligned} D(f \circ g) &= D(g) \cap D(f(g(x))) \\ &= \mathbb{R} \cap (-\infty, -1] \cup [1, \infty) \\ &= (-\infty, -1] \cup [1, \infty). \end{aligned}$$

■

Solution to 19.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x+1}) \\ &= \frac{1}{(\sqrt{x+1})^2 - 1} \\ &= \frac{1}{x+1-1} = \frac{1}{x}.\end{aligned}$$

■

Solution to 20. $D(g \circ f) = D(f) \cap D(g(f(x)))$. Since $(g \circ f)(x) = g(f(x)) = \dfrac{1}{x}$, then $D(g(f(x))) = (-\infty, 0) \cup (0, \infty)$. Since $f(x) = \sqrt{x+1}$, is an even root function, then $x + 1 \geq 0 \Leftrightarrow x \geq -1$. Hence $D(f) = [-1, \infty)$.

$$\begin{aligned} D(g \circ f) &= D(f) \cap D(g(f(x))) \\ &= [-1, \infty) \cap (-\infty, 0) \cup (0, \infty) \\ &= [-1, 0) \cup (0, \infty). \end{aligned}$$

■