VECTORS:

To define a vector write the vector in the form $a = [a_1, a_2, a_3]$ and then click .

Sum of vectors:

Example: $a = [a_1, a_2, a_3]$ press , $b = [b_1, b_2, b_3]$ click , then

write $a + b$ and click

Ex1 :

1- define the vectors $a = [6, 2, 3]$, $b = [-1, 5, 2]$

2- find $a + b$

Dot product of vectors:

To find dot product (scalar product) write ab and then click

Ex2 :

for a and b defined in Ex 1 find $a \cdot b$

Cross product of vectors:

To find cross product : write $a \times b$ and then click

Ex 3:

Find the cross product for : $u = [1, 2, 0]$, $v = [0, 3, 1]$ then verify that it is orthognal to both u and v.

Length of a vector:

To find the length of a vector : define the vector then write $\|a\|$ and then click

Ex 4:

find the length of the vector u and then find the unit vector that has the same direction of u.

Angle between two vectors:

To find the angle between two vectors:

$$A \cdot B = \|A\| \|B\| \cos\theta$$

where θ is the angle between the vectors A and B, you can use the dot product to find the angle between two vectors.

Example *Define* $A = (1, 2, -3)$ *and* $B = (-2, 1, 2)$ *and solve the equation* $\cos\theta = \frac{A \cdot B}{\|A\| \|B\|}$, *to get*

$$\textit{Solution is: } \left\{\theta = \pi - \arccos\left(\frac{1}{7}\sqrt{14}\right)\right\}$$

Apply Evaluate Numerically *to get* $\theta = 2.1347$.

Ex 5:

Find the angle between the two vectors and determine whether they are orthognal:

1- $a = [-5, 3, 7]$, $b = [6, -8, 2]$
2- $u = [-1, 2, 5]$, $v = [3, 4, -1]$

The volume of the parallelepiped spanned by three vectors A, B, and C

$V = |A \cdot (B \times C)|$.

Example:

find the volume of the parallelepiped spanned by $[1, 1, 0]$, $[1, 0, 1]$, and $[0, 1, 1]$.

How to plot Quadric surfaces:

To obtain an implicit plot of an equation involving three variables

1. Enter the equation in your document.
2. From the Plot 3D submenu, choose Implicit.

Ex 6:

Sketch and define then determine the axis and find traces of the following surfaces

1 - $\frac{x^2}{16} - \frac{y^2}{9} + z^2 = -1$

2- $\frac{x^2}{25} + \frac{y^2}{9} - \frac{z^2}{16} = 1$

3- $z = x^2 + \frac{y^2}{2}$

4- $16x^2 + 9y^2 + 16z^2 - 32x - 36y + 36 = 0$

5- $3z = -y^2 + x^2$

6- $z^2 + x^2 - y^2 = 0$

Vector functions and space curves

To define a vector function : **write the vector function in the form** $r(t) = [f(t), g(t), h(t)]$

and then click .

to find derivativeof a vector function : **write** $\frac{d}{dt} r(t)$ **and click** .

to find integralof a vector function : **write** $\int_a^b r(t)\,dt$ **and click** .

to plot a vector function :

1. define the vector function $r(t)$

2. From the Plot3D submenue, choose rectangular

Ex 7:

A) If $r(t) = [1 + t^2, t\exp(-t), \frac{\sin t}{t}]$ B) $r(t) = [t, t^2, t^3]$ C) $r(t) = [t - \frac{3}{2}\sin t, 1 - \frac{3}{2}\cos t, t]$ D) $r(t) = [\cos t, \sin t, \ln t]$

1. find $\int_0^1 r(t)\,dt$
2. find $\frac{d}{dt} r(t)$
3. sketch $r(t)$

function of several variables:

To define afunction of several variables :**write the function** $f(x, y)$, **then click** .

to find the derivative of the function $f(x, y)$: **write** $\frac{\partial}{\partial y} f(x, y)$ **and click**

to find the double integral of the function $f(x, y)$: **write** $\int_a^b \int_c^d f(x, y)\,dydx$ **and click**

Ex 8:

For $f = \frac{x^2 - y^2}{x^2 + y^2}$ find

1. $\frac{\partial^2 f}{\partial x^2}$
2. $\frac{\partial^2 f}{\partial y^2}$
3. $\int_0^1 \int_1^2 f dydx$

To plot a defined function f of two variables

1. Select the function name f or select the expression $f(x, y)$.
2. From the Plot 3D submenu, choose Rectangular.

Ex 9:

Sketch the function A) $f(x, y) = \frac{\sin x \sin y}{xy}$ B) $f(x, y) = \frac{-3y}{x^2 + y^2 + 1}$

Gradiant of a function of several variables:

To evaluate the gradiant of a function of several variables: define the function f, then write∇f and click

To plot a gradient field

1. Type an expression $f(x, y)$.
2. Leave the insertion point in the expression, and from the Plot 2D submenu, choose

Gradient.

Ex 10 :

If A) $f = xy^2z^3$,B) $h(x,y) = xy\sin xy$, find the gradiant of the function and plot its graph.

Vector field

To find the divergent of a vector field :**define the vector field** ,**then write** $\nabla \cdot F$ **and click**

to find the curl of a vector field : **define the vector field** ,**then write** $\nabla \times F$ **and click**

To plot a two-dimensional vector field

1. Type a pair of two-variable expressions
2. Leave the insertion point in the vector, and from the Plot 2D submenu, choose Vector Field

Ex 11:

For A) $F = \left(\frac{1}{\sqrt{\left(1 + |y^2 - t^2|^2\right)}}, \frac{y^2 - t^2}{\sqrt{\left(1 + |y^2 - t^2|^2\right)}} \right)$ B) $F = [\ln(1 + y^2), \ln(1 + x^2)]$, plot F

To plot a three-dimensional vector field

1. Type three three-variable expressions, representing the *x*-, *y*-, and *z*-components of a vector field, into a vector.
2. Leave the insertion point in the vector.
3. From the Plot 3D submenu, choose Vector Field.

To change the view

1. Click the frame until a small box appears in the upper-right corner of the frame.
2. With the left mouse button held down, rotate the plot.

Ex 12:

For A) $F = [xz, 2xz, zy]$ B) $F = [\frac{y}{z}, -\frac{x}{z}, \frac{z}{4}]$ C) $F = [yz, xz, xy]$ find

1. $\nabla \cdot F$

2. $\nabla \times F$

3. sketch F.