Additional Elements of Math. Models

- Transport rate equation

  Transports phenomena (mass, energy, momentum)

  - Q (ex. supplied by team) = $UA_t(T_{st} - T)$

  - mass $N_A = K_c a\,(C_A - C_{Ai})$

  - Kinetic rate equation $-r_A = kC_A = k_o e^{(-E/RT)}\,C_A$

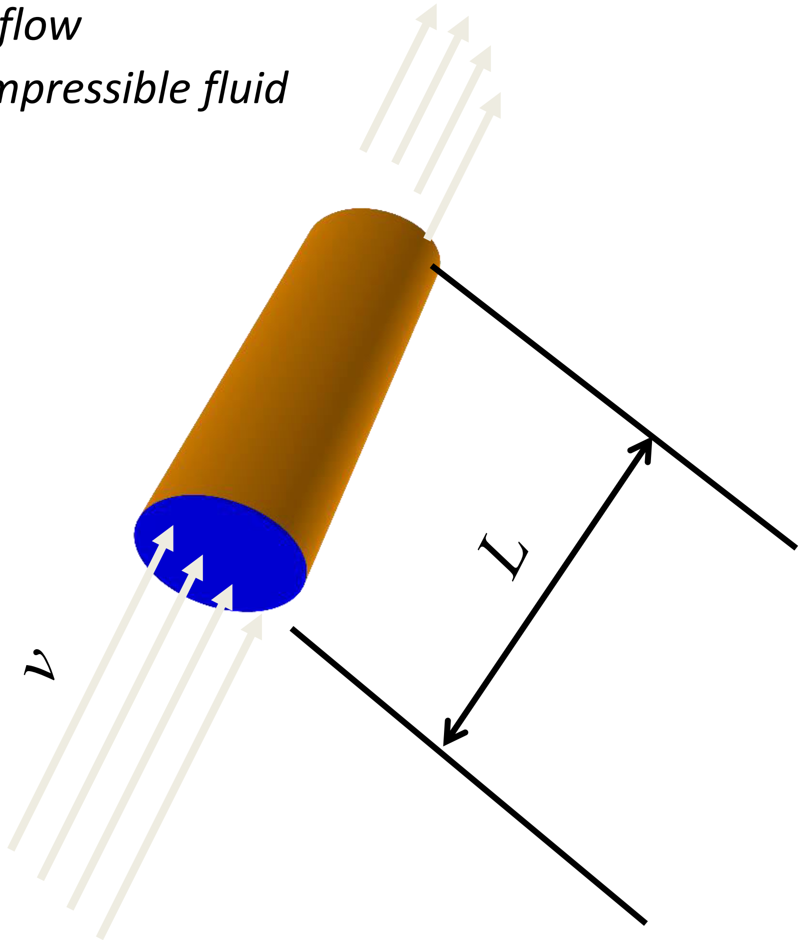
- Phase Equilibrium

- Equation of state

- Dead time

Pure time-delay systems

Plug flow
Incompressible fluid



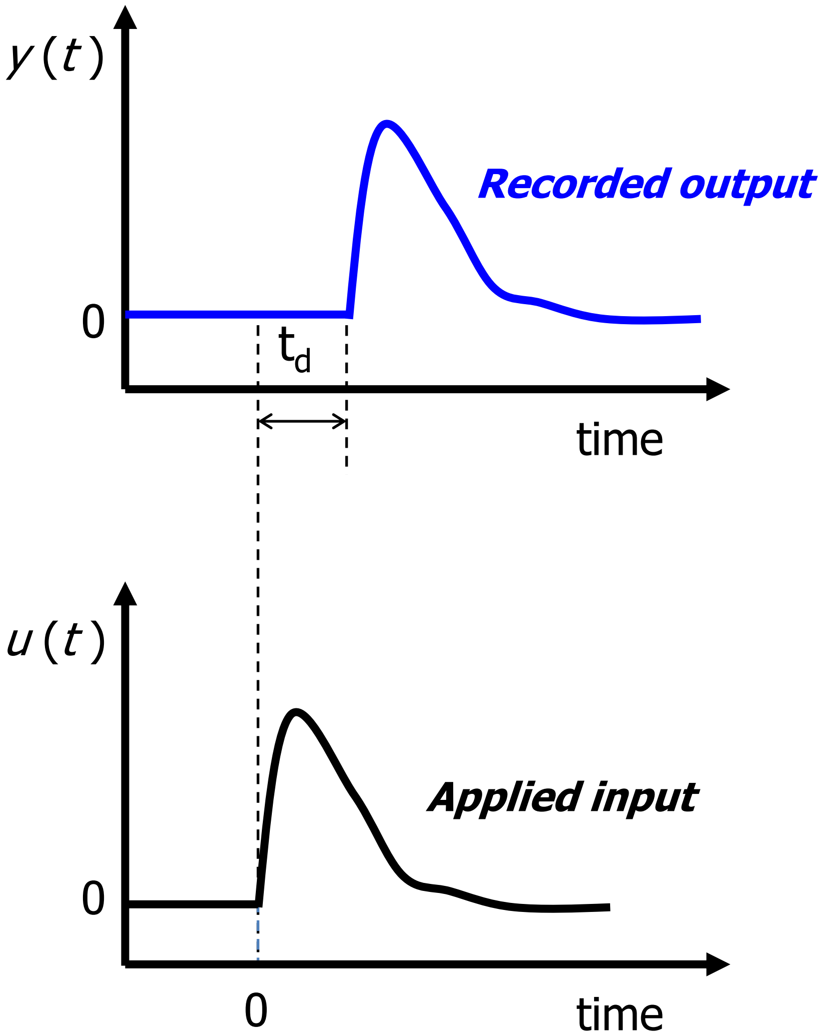
- Many real systems do not react *immediately* to excitation (as first order systems instead do)
- The time needed to “transport” a fluid property change from the inlet to the outlet is:

$$t_d = \frac{L}{v}$$

: ***dead time*** or ***time delay***

Examples: transportation lags (e.g. due to pipe length, to recycle, …); measurement lags (e.g. gaschromatographs)

Pure time-delay systems (cont'd)



The process output is simply shifted by t_d units in time with respect to the input

Models

Time domain :

$$y(t) = \begin{cases} 0 \quad , & t < t_d \\ x(t - t_d), & t \geq t_d \end{cases}$$

Degree of Freedom (f) & Process Control

- f is the number of independent variables that must be specified in order to define the process completely.

$$f = V - E$$

$f = 0$ Exact solution (unique sol.)

$f > 0$ Multiple solution

$f < 0$ ore we No solution

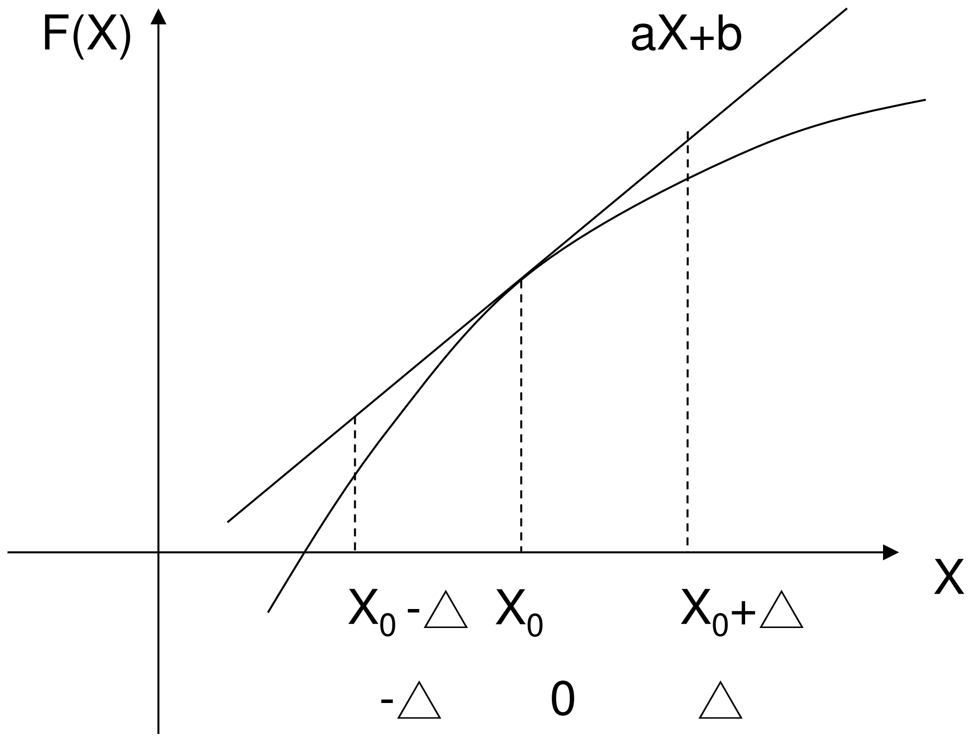
objective is to make $f = 0$

Number of Variables is fixed. We need to add f equations for exact solution. Sources of equations are:

1- External world

2- Control system

Linearization of a Function



Linearization

- First order Taylor series expansion

1. Function of one variable

$$f(x) \approx f(x_s) + \frac{\partial f(x_s)}{\partial x}(x - x_s)$$

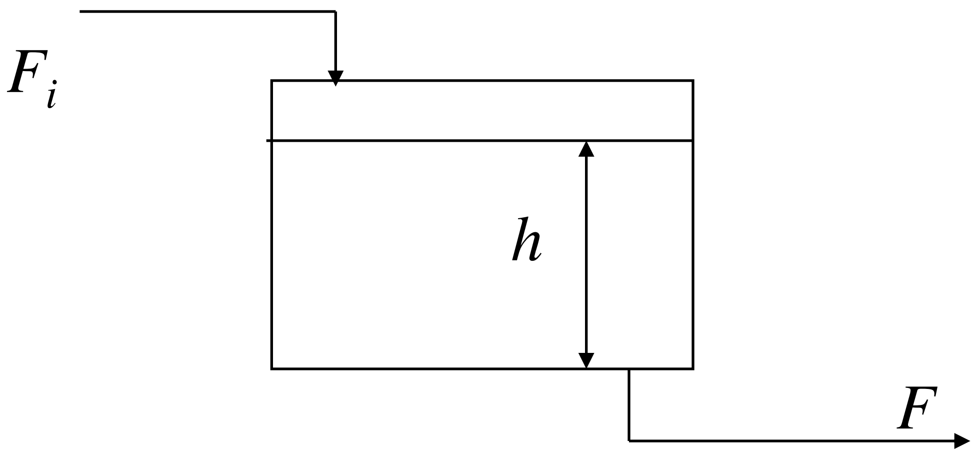
2. Function of two variables

$$f(x,u) \approx f(x_s,u_s) + \frac{\partial f(x_s,u_s)}{\partial x}(x - x_s) + \frac{\partial f(x_s,u_s)}{\partial u}(u - u_s)$$

3. ODEs

$$\dot{x} = f(x) \approx f(x_s) + \frac{\partial f(x_s)}{\partial x}(x - x_s)$$

Example, Liquid storage



$$\rho A \frac{dh}{dt} = \rho F_i - \rho F = \rho F_i - \beta h$$

$$\frac{\rho A}{\beta} \frac{dh}{dt} = \frac{\rho}{\beta} F_i - h$$

$$\tau \frac{dh}{dt} + h = K_p F_i$$

$$\tau \frac{dh'}{dt} + h' = K_p F_i{}'$$