0.4. Trigonometric Functions :

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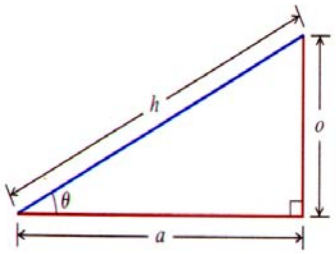

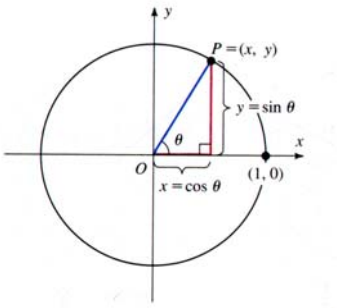

Unit circle

1. $\sin\theta = \frac{o}{h}$
2. $\cos\theta = \frac{a}{h}$
3. $\tan\theta = \frac{o}{a}$
4. $\sec\theta = \frac{h}{a}$
5. $\csc\theta = \frac{h}{o}$
6. $\cot\theta = \frac{a}{o}$

- **Some trigomomertic identities.**

$\sin x = \frac{1}{\csc x}$ & $\csc x = \frac{1}{\sin x}$	$\cos x = \frac{1}{\sec x}$ & $\sec x = \frac{1}{\cos x}$
$\tan x = \frac{1}{\cot x}$ & $\cot x = \frac{1}{\tan x}$	$\tan x = \frac{\sin x}{\cos x}$ & $\cot x = \frac{\cos x}{\sin x}$
$\sin^2 x + \cos^2 x = 1$	$\sec^2 x - \tan^2 x = 1$
$1 + \cot^2 x = \csc^2 x$	$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$	$\tan(-x) = -\tan x$
$\sec(-x) = \sec x$	$\cot(-x) = -\cot x$
$\csc(-x) = -\csc x$	$\sin 2x = 2\sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
$\sin^2 x = \frac{1 - \cos 2x}{2}$	$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$	$\sin(\frac{\pi}{2} - x) = \cos x$
$\cos(\frac{\pi}{2} - x) = \sin x$	$\tan(\frac{\pi}{2} - x) = \cot x$
$\sec(\frac{\pi}{2} - x) = \csc x$	$\csc(\frac{\pi}{2} - x) = \sec x$
$\cot(\frac{\pi}{2} - x) = \tan x$	$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	undefined	0

Example 4.4. Derive the identities:

1) $\sin 2x = 2\sin x \cos x$

Solution: Set $y = x$ in the identit

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

Then

$$\sin(2x) = \sin x \cos x + \cos x \sin x$$
$$= 2\sin x \cos x$$

2) $\cos 2x = \cos^2 x - \sin^2 x$

Solution: Set $y = x$ in the identit

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

Then

$$\cos(2x) = \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x$$

3) $\cos^2 x = \frac{1+\cos 2x}{2}$ and $\sin^2 x = \frac{1-\cos 2x}{2}$

Solution: We **have**

$$\cos(2x) = \cos^2 x - \sin^2 x$$

and

$$\sin^2 x = 1 - \cos^2 x$$

Then

$$\begin{aligned}\cos(2x) &= \cos^2 x - \sin^2 x \\ &= \cos^2 x - (1 - \cos^2 x) \\ &= \cos^2 x - 1 + \cos^2 x \\ &= 2\cos^2 x - 1\end{aligned}$$

Thus

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Similarly, we can prove $\sin^2 x = \frac{1 - \cos 2x}{2}$.

Remarks: (1) To convert the radians measure x to degrees, we apply

$$x = x\,\frac{180^\circ}{\pi}$$

(1) To convert the degree measure x to radians, we apply

$$x = x\,\frac{\pi}{180^\circ}$$

Exercises 0.4

1) Convert the given radians measure to degrees.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{4\pi}{3}$

Sol: To convert to degrees

$$\frac{4\pi}{3} = \frac{4\pi}{3}\frac{180^\circ}{\pi} = 240^\circ$$

2) Convert the given degrees measure to radians.

(a) 180° (b) 270° (c) 120° (d) 30°

Sol: (b) To convert to radians

3) Prove the given trigonometric identities:

1) $\sin(x - y) = \sin x \cos y - \cos x \sin y$

2) $\cos(x - y) = \cos x \cos y + \sin x \sin y$