3. Array and Matrix Operations

Almost anything you learned about in your linear algebra classMatlab has a command to do. Here is a brief summary of the most useful ones for physics.
In MATLAB matrices are defined inside a pair of square braces ([]). Punctuation marks of a comma (,), and semicolon (;) are used as a row separator and column separator, respectfully.
Note: you can also use a space as a row separator, and a carriage return (the enter key) as a column separator as well.

3.1 Simple Arrays and Operations

Below are examples of how a matrix can be created in MATLAB, (*command window*).

Vectors come in two flavors - row vectors and column vectors. In either case they are lists of numbers separated by either commas or spaces. The number of entries is known as the "length" of the vector and the entries are called "elements" or "components" of the vector. The entries must be enclosed by square brackets.

```
>> V = [1 3, sqrt(5)]
V =
    1.0000    3.0000    2.2361

>> V1 = V'       % Transpose
V1 =
    1.0000
    3.0000
    2.2361

>> sum(V1)
ans =
    6.2361

>> A = [1 2 3;4 5 6;7 8 0]
A =
    1    2    3
    4    5    6
    7    8    0
```

Semicolons are used to separate the rows of a matrix. Note that the entries of the matrix must be separated by a single space.
The components of matrices can be manipulated in several ways. For example

13

```
>> A(2,3) % select a single entry of A
ans =
        6

>> A(2,:)
ans =
      4      5       6

>> b = A(:,1)
b =
      1
      4
      7

>>A(1:2,2:3)   %select a submatrix of A
ans=
      2 3
      5 6

>>A([1 3],[1 3])   %another way to select a submatrix of A
  ans=
      1 3
      7 9

>>A(2,2)=tan(7.8);   %assign a new value to an entry of A
```

Arrays operators

When they are taken away from the world of linear algebra, matrices become two-dimensional numeric arrays. Arithmetic operations on arrays are done element by element. This means that addition and subtraction are the same for arrays and matrices, but that multiplicative operations are different. MATLAB uses a dot, or decimal point, as part of the notation for multiplicative array operations.

The list of operators includes

- $+$ Addition
- $-$ Subtraction
- $.*$ Element-by-element multiplication
- $./$ Element-by-element division
- $.\backslash$ Element-by-element left division
- $.\hat{}$ Element-by-element power
- $.'$ Unconjugated array transpose

14

Example: find the difference between A*A and A*.A

```
A*A   =                        A.*A   =
      30      36      15               1       4       9
      66      81      42              16      25      36
      39      54      69              49      64       0
```

Special Matrices and Vectors

MATLAB software provides four functions that generate basic matrices.

Function	Description
zeros	All zeros
ones	All ones
rand	Uniformly distributed random elements
randne	Normally distributed random elements
Eye(n)	Returns the n-by-n identity matrix.

Here are some examples:

```
Z = zeros(2,4)
Z =
     0       0       0       0
     0       0       0       0

F = 5*ones(3,3)
F =
     5       5       5
     5       5       5
     5       5       5

N = fix(10*rand(1,10))
N =
     9       2       6       4       8       7       4       0       8       4

R = randn(3,4)
R =
    0.5377      0.8622     -0.4336      2.7694
    1.8339      0.3188      0.3426     -1.3499
   -2.2588     -1.3077      3.5784      3.0349
```

Basic Operations and Descriptive Statistics

The Statistics library provides fundamental statistical operations such as minimum, maximum, mean, variance, and standard deviation. Here is a table of some basic Operations:

Function	Description
brush	Interactively mark, delete, modify, and save observations in graphs
cumprod	Cumulative product
cumsum	Cumulative sum
linkdata	Automatically update graphs when variables change
prod	Product of array elements
sort	Sort array elements in ascending or descending order
sortrows	Sort rows in ascending order
sum	Sum of array elements
corrcoef	Correlation coefficients
cov	Covariance matrix
max	Largest elements in array
mean	Average or mean value of array
median	Median value of array
min	Smallest elements in array
mode	Most frequent values in array
std	Standard deviation
var	Variance

Example

```
Let A = [1 2 3; 3 3 6; 4 6 8; 4 7 7]; and B = [2,7,8,3];

prod(B)
ans =
   336

mean(A)   % mean of each column
ans =
    3.0000    4.5000    6.0000

mean(A,2)  % mean of each row
ans =
    2.0000
    4.0000
    6.0000
    6.0000
```

3.2 Polynomials

Polynomial functions are located in the MATLAB `polyfun` directory. For a complete list, brief descriptions, and links to reference pages, type:

```
help polyfun
```
The following table lists the MATLAB polynomial functions.

Function	Description
conv	Multiply polynomials
deconv	Divide polynomials
poly	Polynomial with specified roots
polyder	Polynomial derivative
polyfit	Polynomial curve fitting
polyval	Polynomial evaluation
polyvalm	Matrix polynomial evaluation
residue	Partial-fraction expansion (residues)
roots	Find polynomial roots

Representing Polynomials

MATLAB software represents polynomials as row vectors containing coefficients ordered by descending powers. For example, consider the equation

$$p(x) = x^3 - 2x - 5$$

To enter this polynomial into MATLAB, use
```
      p = [1 0 -2 -5];
```

Evaluating Polynomials

The polyval function evaluates a polynomial at a specified value. To evaluate p at $x = 5$, use

```
polyval(p,5)
ans =
110
```

It is also possible to evaluate a polynomial in a matrix sense.
For example, create a square matrix **X** and evaluate the polynomial p at X:

```
X = [2 4 5; -1 0 3; 7 1 5];
Y = polyvalm(p,X)
```

```
Y =
   377    179    439
   111     81    136
   490    253    639
```

Roots of Polynomials

The roots function calculates the roots of a polynomial:

```
r = roots(p)

r =
    2.0946
   -1.0473 + 1.1359i
   -1.0473 - 1.1359i
```

By convention, the MATLAB software stores roots in column vectors. The function poly returns to the polynomial coefficients:

```
p2 = poly(r)

p2 =
     1.0000   -0.0000   -2.0000   -5.0000
```

Derivatives

The polyder function computes the derivative of any polynomial. To obtain the derivative of the polynomial `p = [1 0 -2 -5];`

```
q = polyder(p)

q =
     3     0    -2
```

polyder also computes the derivative of the product or quotient of two polynomials. For example, create two polynomials a and b:

```
a = [1 3 5];
b = [2 4 6];
```

Calculate the derivative of the product a*b by calling polyder with a single output argument:

```
c = polyder(a,b)
```

```
c =
     8    30    56    38
```

Calculate the derivative of the quotient a/b by calling `polyder` with two output arguments:

```
[q,d] = polyder(a,b)
q =

    -2    -8    -2

d =
     4    16    40    48    36
```

q/d is the result of the operation.

Convolution

Polynomial multiplication and division correspond to the operations convolution and deconvolution. The functions conv and deconv implement these operations.
Consider the polynomials $a(t) = t^2 + 2t + 3$ and $b(t) = 4t^2 + 5t + 6$.
To compute their product,

```
a = [1 2 3]; b = [4 5 6];
c = conv(a,b)

c =
     4    13    28    27    18
```

Use deconvolution to divide $a(t)$ back out of the product:

```
[q,r] = deconv(c,a)
q =
     4     5     6
r =
     0     0     0     0     0
```

Partial Fraction Expansions

The command `residue` finds the partial fraction expansion of the ratio of two polynomials. This is particularly useful for applications that represent systems in transfer function form. For polynomials b and a, if there are no multiple roots,

19

$$\frac{b(s)}{a(s)} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2} + \cdots + \frac{r_n}{s - p_n} + k_s$$

where r is a column vector of residues, p is a column vector of pole locations, and k is a row vector of direct terms. Consider the transfer function

$$\frac{-4s + 8}{s^2 + 6s + 8}$$

```
b = [-4 8];
a = [1 6 8];
[r,p,k] = residue(b,a)

r =
   -12
     8

p =
    -4
    -2

k =
    []
```

That is $\frac{-4s+8}{s^2+6s+8} = \frac{-12}{s+4} + \frac{8}{s+2} + 0$

Given three input arguments (r, p, and k), residue converts back to polynomial form:

```
[b2,a2] = residue(r,p,k)
b2 =
    -4      8

a2 =
     1     6      8
```

Characteristic Polynomials

The poly function also computes the coefficients of the characteristic polynomial of a matrix:

```
A = [1.2 3 -0.9; 5 1.75 6; 9 0 1];
poly(A)
```

```
ans =
     1.0000   -3.9500   -1.8500  -163.2750
```

3.3   MATLAB Linear Algebra Functions

The MATLAB `matfun` directory contains linear algebra functions. For a complete list, brief descriptions, and links to reference pages, type:

```
>> help matfun
```

The following table lists the some MATLAB linear algebra functions by category.

Function	Description	Function	Description
norm	Matrix or vector norm	inv	Matrix inverse
rank	Matrix rank	subspace	Angle between two subspaces
det	Determinant	linsolve	Solve a system of linear equations
trace	Sum of diagonal elements	lu	LU factorization
null	Null space	chol	Cholesky factorization
orth	Orthogonalization	eig	Eigenvalues and eigenvectors
rref	Reduced row echelon form	expm	Matrix exponential
pinv	Pseudoinverse	funm	Evaluate general matrix function

Matrix Factorizations
The Matrix Factorizations library provides the following blocks for factoring various kinds of matrices:

- Cholesky Factorization
- LDL Factorization
- LU Factorization
- QR Factorization
- Singular Value Decomposition

We will discuss and show example for LU Factorization only. The `lu` function expresses a matrix **A** as the product of two essentially triangular matrices, one of them a permutation of a lower triangular matrix and the other an upper triangular matrix. The factorization is often called the *LU*, or sometimes the *LR*, factorization. **A** can be rectangular. Here is an example for the LU Factorization: let

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & 6 \\ 2 & -1 & 3 \end{bmatrix}$$

The MATLB command is

```
A=[1 -2 3;4 0 6; 2 -1,3]
[L,U,P] = lu(A)
```

The output are
```
L =
     1.0000           0           0
     0.2500      1.0000           0
     0.5000      0.5000      1.0000


U =
     4.0000           0      6.0000
          0     -2.0000      1.5000
          0           0     -0.7500


P =
     0      1      0
     1      0      0
     0      0      1
```

Where

L	Is either a unit lower triangular matrix, or else the product of a unit lower triangular matrix with `P'`.
U	Upper triangular matrix that is a factor of `A`.
P	Row permutation matrix satisfying the equation `L*U = P*A`, or `L*U = P*A*Q`.

Linear system of equations

A linear system can be solved in more then one comandes for example: The system

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$$

can be solved as:

```
>> A = [1 0 3;0 5 6;7 8 0];    b = [5;-2;3];    x = inv(A)*b
x =
     2.1765
    -1.5294
     0.9412
Or
>> y=linsolve(A,b)
y =
     2.1765
    -1.5294
     0.9412
```

Exercises

1. Create the 3×3 matrix whose entries in the first row and first column are all equal to 1 and all the other entries are equal to e.

2. Generate a row vector of the operations **(i)** $y = \sin(x)$, **(ii)** $z = x^2$, **(iii)** $w = 1/x$, where x is even integers $0 < x < 10$.

3. Enter a matrix v consisting of 9 equally spaced entries between 0 and 2π. Compute $\cos v$.

4. Generate the 1×10 row vector v whose i-th component is $\cos(i\pi/4)$.

5. Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 3 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

    Calculate
    a. $C = A + B$, $D = A - B$ and $E = AB$
    b. $F = B^t A^t$
    c. Evaluate the determinant of A, B and C
    d. Evaluate the eigenvals and the eigenvectors of the matrix A, B and C

6. Solve the following linear system of equations
$$3x - 6y + 2z = 15$$
$$-4x + y - z = -2$$
$$x - 3y + 7z = 22$$

7. Use hep to find minimum and maximum values for each row of
$$P = [0\ 1\ 2;\ -1\ -2\ -0.5]$$

8. Find the roots of the polynomials
    a) $P(x) = x^5 - x^4 - x^3 + x^2 - 2x + 2$
    b) $P(x) = x^5 - 2x^3 - 12x^2 + 9x$

9. Solve the following equations

    a) $\begin{bmatrix} 2 & 1 & -3 \\ -1 & 3 & 2 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 12 \\ 0 \end{bmatrix}$
    b) $\begin{bmatrix} 0.1 & -0.6 & 1 \\ -2 & 8 & 0.3 \\ 1 & 6 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

10. Let $\vec{a} = [1\ 2\ 3]$ and $\vec{b} = [4\ 5\ 6]$, find $a \times b^T$ and $a \cdot b$.

11. Find the volume of the parallelepiped with edges $\vec{a} = (0,0,1)$, $\vec{b} = (0,2,2)$, and $\vec{c} = (3,3,3)$.

12. For a given even number N, find the sum of the series
$(1 - 1/2) + (1/3 - 1/4) + (1/5 - 1/6) \cdots$, *(use input command for N).*

13. Let A be a random matrix generated by rand(8). Find the maximum values:
 (a) In each column **(b)** in each row **(c)** overall and **(d)** find the row and column
   indices of these maximum values.

14. Suppose A is any matrix. What dose this statemet do?
   A(1:size(A,1)+1:end)

15. Given $P(x) = 2.2x^5 - 3.1x^3 + 0.13$ and $Q(x) = x^2 + 3x - 5$ find

 a) $P(0.5)$ **b)** all roots of P

 c) $\dfrac{d^2P}{dx^2}$ **d)** $P \times Q$ and $\dfrac{P(x)}{Q(x)}$

16. Find the partial fractions and the residual for ratio $\dfrac{2x^2+5x+1}{x^2+2x-15}$